Examination 2

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the

<div style="border:1px solid">**comments in boxes.**</div>

1. [15] Prove that $(\forall x)Px \wedge (\forall x)Qx$ follows from $(\forall x)(Px \wedge Qx)$

2. [10] Using induction, prove that for $n \geq 0$, $\displaystyle\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$

3.   **a [5]** Prove for any set A, that $A = A \cap A$.

   b. [5] Prove for any set A, that $A = A \cup A$.

   c. [10] Using parts a. and b. to prove that $A \cup B = A \cap B$ if and only if $A = B$.

4. [15]. Given sets $A, B,$ and C and relations R, between A and B, and S, between B and C, prove that $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.

5. [20]. Prove that if R is a partial order on a set A, then R^{-1} is also a partial order on A.

6. Consider relations R and S on a set A . Prove or disprove with a simple counter example each of the following:

   a. [10] If R and S are symmetric, then $R \sim S$ is symmetric.

   b. [10] If R and S are antisymmetric, then $R \sim S$ is antisymmetric.

   c. [10] If R and S are transitive, then $R \sim S$ is transitive.