1. [15] Prove that \((\exists x)(P_x \land A)\) follows from \((\exists x)(P_x \land A)\)

- \(\{P\}\) (1). \((\exists x)(P_x \land A)\)  
- \(\{P\}\) (2). \(P_a \land A\)  
- \(\{P\}\) (3). \(P_a\)  
- \(\{P\}\) (4). \((\exists x)P_x\)  
- \(\{P\}\) (5). \(A\)  
- \(\{P\}\) (6). \((\exists x)(P_x \land A)\)  

2. [10] For any sets \(A\) and \(B\), prove that \(A \sim (A \sim B) = A \cap B\).

\[
\begin{align*}
x &\in A \sim (A \sim B) \\
\Rightarrow x &\in A \land \sim (x \in A \sim B) \\
\Rightarrow x &\in A \land \sim (x \in A \land x \notin B) \\
\Rightarrow x &\in A \land (x \notin A \lor x \in B) \\
\Rightarrow (x \in A \land x \notin A) \lor (x \in A \land x \in B) \\
\Rightarrow x &\in A \land x \in B \\
\Rightarrow x &\in A \cap B.
\end{align*}
\]

\[
\begin{align*}
x &\in A \cap B \\
\Rightarrow x &\in A \land x \in B \\
\Rightarrow x &\in A \land x \in B \\
\Rightarrow x &\in A \land (x \notin A \lor x \in B) \\
\Rightarrow x &\in A \land \sim (x \in A \land x \notin B) \\
\Rightarrow x &\in A \land \sim (x \in A \sim B) \\
\Rightarrow x &\in A \sim (A \sim B).
\end{align*}
\]
3. [20] Using induction prove for \( n \geq 2 \), that 
\[
\prod_{k=2}^{n}(1-\frac{1}{k^2}) = \frac{n+1}{2n}.
\]

For \( n = 2 \), we have
\[
\prod_{k=2}^{2}(1-\frac{1}{k^2}) = \prod_{k=2}^{2}(1-\frac{1}{k^2})
= 1-\frac{1}{4}
= \frac{3}{4}
= \frac{2+1}{2.2}
= \frac{n+1}{2n}.
\]

Now assume for some \( n \geq 2 \), that \( \prod_{k=2}^{n}(1-\frac{1}{k^2}) = \frac{n+1}{2n} \). We then have
\[
\prod_{k=2}^{n+1}(1-\frac{1}{k^2}) = \prod_{k=2}^{n}(1-\frac{1}{k^2})(1-\frac{1}{(n+1)^2})
= \frac{n+1}{2n} (1-\frac{1}{(n+1)^2})
= \frac{n+1}{2n} (n+1)^2 - 1
= \frac{n+1}{2n} \frac{n(n+2)}{(n+1)^2}
= \frac{n+2}{2(n+1)}
= \frac{(n+1)+1}{2(n+1)}.
\]

4. [10] For any sets \( A \) and \( B \), prove that \( P(A \cap B) = P(A) \cap P(B) \).

\[
X \in P(A \cap B) \implies X \subseteq A \cap B \implies X \subseteq A \land X \subseteq B \implies X \in P(A) \land X \in P(B) \implies X \in P(A) \cap P(B).
\]
5. [10] Given a set $A$ and two symmetric relations $R$ and $S$ on $A$, prove or disprove with a simple counter-example: $R \circ S$ is symmetric.

$R \circ S$ need not be symmetric. Let $A = \{1,2,3\}$, $S = \{(1,2),(2,1)\}$, and $R = \{(2,3),(3,2)\}$. Then $R \circ S = \{(3,1)\} \in R \circ S$ but $(3,1) \in R \circ S$.

6. [20] Consider the relation $R$ on $\mathbb{Z}$, the set of integers: $R = \{(x, y): x + y \text{ is even}\}$. Prove that $R$ is an equivalence relation.

We seek to show $R$ is reflexive, symmetric, and transitive. To that end, consider any $x \in \mathbb{Z}$. Since $x + x = 2x$ is even, $(x, x) \in R$ and $R$ is reflexive. If $(x, y) \in R$ then $x + y$ is even, thus $y + x$ is even, $(y, x) \in R$, and $R$ is symmetric. Lastly, suppose $(x, y) \in R$ and $(y, z) \in R$ thus both $x + y$ and $y + z$ are even. The sum $x + z + 2y$ is even as well as $2y$, so the difference $x + z = (x + z + 2y) - 2y$ is even so $(x, z) \in R$ and $R$ is transitive. We conclude that $R$ is an equivalence relation.