Examination 2

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [15] Using induction, prove that for $n \geq 1$, $n^3 + 2n$ is an integral multiple of 3
(i.e. $\forall n \in \mathbb{Z}((n \geq 1) \Rightarrow \exists k \in \mathbb{Z}(n^3 + 2n = 3k))$.

2. [10] Using induction, prove that for $n \geq 1$, $\sum_{k=1}^{n}(4k-3) = n(2n-1)$.

3. a[5]Prove for any sets $A, B,$ and $C,$ that $(A \cup B) \sim C = (A \sim C) \cup (B \sim C)$.

b.[10]Using induction and part a, prove for $n \geq 1$, all sets $A_1, A_2, \ldots, A_n$, and all C:

$$(\bigcup_{i=1}^{n} A_i) \sim C = \bigcup_{i=1}^{n}(A_i \sim C).$$

4. [15] Using induction, prove a generalized distributivity law for sets – that is, for $n \geq 1$ and all sets A and $B_1, B_2, \ldots, B_n$,

$$A \cup \bigcap_{i=1}^{n} B_i = \bigcap_{i=1}^{n}(A \cup B_i).$$

(Recall that $\bigcap_{i=1}^{n+1} B_i = (\bigcap_{i=1}^{n} B_i) \cap B_{n+1}$.)

5. [10] For all sets $A, B, C,$ and D, prove that $(A \cap B) \times (C \cap D) \subseteq (A \times C) \cap (B \times D)$.

6. a [10]. Either prove or give a simple counterexample. Given symmetric relations R and S on a set A, the composition $S \circ R$ is symmetric. (If you present a counterexample, present the relations as specific sets of ordered pairs rather using matrices or graphs.)

b [10]. Either prove or give a simple counterexample. Given antisymmetric relations R and S on a set A, the difference $R \sim S$ is antisymmetric. (If you present a counterexample, present the relations as specific sets of ordered pairs rather using matrices or graphs.)