Examination 2

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the **comments in boxes**

1. [10] For fixed real numbers a and b, consider the iteratively defined sequence:

$$s_0 = a$$

$$s_n = 2s_{n-1} + b, \; for\, n \geq 1.$$

Using induction, prove that for $n \geq 0, s_n = 2^n a + (2^n - 1)b$.

2. [10] Using induction, prove that for $n \geq 4$, $n! > 2^n$.

3. [10]Prove for any sets A, B, and C, that if $A \subseteq C \cup B$ then $A \sim C \subseteq B$.

4. [10]. Given sets A, B, and C , prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

5. Show these two definitions of antisymmetry for a relation R on a set A are equivalent:

a. $\forall x, y \in A : ((x, y) \in R \wedge x \neq y) \Rightarrow (y, x) \notin R$.

b. $\forall x, y \in A : ((x, y) \in R \wedge (y, x) \in R) \Rightarrow x = y$.

(Hint: Ignore the universal quantifier, let $P = "(x, y) \in R", Q = "(y, x) \in R"$, and $E = "x = y"$. Use simple logical identities to convert one to the other.)

6. For these problems either prove the claim or give a simple counterexample. If you present a counterexample, present the relations as specific sets of ordered pairs rather than using matrices or graphs. For assume R and S are relations on a set A and $R \subseteq S$.

a [10]. If R is transitive then S is transitive.
b [10]. If S is antisymmetric then R is antisymmetric. (Note the reversal of the order from part a.)
c [10]. If R is transitive then $R \circ R$ is transitive.
d [10]. If R is symmetric then $R \circ R$ is symmetric.