1. [10] For fixed real numbers \(a\) and \(b\), with \(a \neq 1\), define
\[
x_0 = 0,
\]
and for \(k = 1, 2, \ldots\),
\[
x_k = a x_{k-1} + b.
\]
Using induction, prove that for \(k \geq 0\), \(x_k = \frac{a^k - 1}{a - 1} b\).

2. [10] Prove for any sets \(A, B,\) and \(C\), that if \(A \subseteq B\) then \(C \sim B \subseteq C \sim A\).

3.a [10] You are given a relation \(R\) on a set \(A\). Prove that \(R\) is antisymmetric if and only if \(R \cap R^{-1} \subseteq I\).

b [10] You are given relations \(R\) and \(S\) on a set \(A\). Prove that
\[
(R \cap S) \circ (R \cap S) \subseteq (R \circ R) \cap (S \circ S)\]

4. [15]. You are given a relation \(R\) on a set \(A\). Using induction, prove that if \(R\) is transitive then for \(k \geq 1\).
\[
R^k \subseteq R.
\]
(Recall \(R^1 = R\) and for \(k \geq 1\), \(R^{k+1} = R \circ R^k\).)

5. [15]. A relation \(R\) on a set \(A\) is countertransitive if and only if for all \(x, y, z \in A\),
\[
((x, y) \in R \wedge (y, z) \in R) \Rightarrow (z, x) \in R.
\]
a. Prove that if \(R\) is symmetric and transitive then \(R\) is also countertransitive.

b. Prove that if \(R\) is countertransitive and reflexive then \(R\) is also symmetric.

c. Prove that if \(R\) is countertransitive and reflexive then \(R\) is also transitive.