1	10	
2	10	
3	20	
4	15	
5	15	
Total	70	

Name ______________________________

Examination 2

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose **comments in boxes.**

1. [10] For fixed real numbers a and b, with $a \neq 1$, define

$$x_0 = 0,$$

and for $k = 1,2,\ldots$

$$x_k = a\, x_{k-1} + b.$$

Using induction, prove that for $k \geq 0$, $x_k = \dfrac{a^k - 1}{a-1} b.$

2. [10] Prove for any sets $A, B,$ and $C,$ that if $A \subseteq B$ then $C \sim B \subseteq C \sim A$.

3.a [10] You are given a relation R on a set A. Prove that R is antisymmetric if and only if $R \cap R^{-1} \subseteq I.$

b [10] You are given relations R and S on a set A. Prove that

$$(R \cap S) \circ (R \cap S) \subseteq (R \circ R) \cap (S \circ S)$$

4. [15]. You are given a relation R on a set A. Using induction, prove that if R is transitive then for $k \geq 1$.

$$R^k \subseteq R.$$

(Recall $R^1 = R$ and for $k \geq 1$, $R^{k+1} = R \circ R^k$.)

5. **[15]**. A relation R on a set A is *countertransitive* if and only if for all $x, y, z \in A$, $((x, y) \in R \wedge (y, z) \in R) \Rightarrow (z, x) \in R$.

a. Prove that if R is symmetric and transitive then R is also countertransitive.

b. Prove that if R is countertransitive and reflexive then R is also symmetric.

c. Prove that if R is countertransitive and reflexive then R is also transitive.