1	10	
2	10	
3	15	
4	15	
5	20	
Total	70	

Name ______________________________

Examination 2

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the **comments in boxes.**

1. [15] Prove that $(\exists z)Qz$ follows from $(\forall x)(\sim Px \wedge (\exists y)(\sim Qy \Rightarrow Px))$.

2. [10] Assuming $\lambda \neq 0$ and $\lambda \neq 1$ and using induction, prove that for $n \geq 0$, $\sum_{k=0}^{n} \lambda^k = \frac{\lambda^{n+1}-1}{\lambda - 1}$.

3. [10] Prove for any sets $A, B,$ and $C,$ that $((A \cup B) \sim C) \cup (A \cap C) = A \cup (B \sim C)$. (Hint: You might want start by proving $((A \cup B) \sim C) \cup (A \cap C) \subseteq A \cup (B \sim C)$.)

4. [15] Using induction, prove a generalized distributivity law for sets – that is, for $n \geq 1$ and all sets A and $B_1, B_2, ..., B_n$,

$$A \cup \bigcap_{i=1}^{n} B_i = \bigcap_{i=1}^{n} (A \cup B_i).$$

(Recall that $\bigcap_{i=1}^{n+1} B_i = (\bigcap_{i=1}^{n} B_i) \cap B_{n+1}$.)

5. [15]. Given a relation R on a set A, prove the relation

$$S = R \cup R^{-1} \cup I$$

is reflexive and symmetric.

6. [20]. A relation R on a set A is *unsymmetric* if and only if it is not symmetric. Prove that if R is a reflexive, unsymmetric, and transitive relation on a set A, then R^{-1} is also a reflexive, unsymmetric, and transitive relation on A.