Final Examination

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the

$\boxed{\textbf{comments in boxes}}$

1. [10] Use a truth table to determine for which truth values of p, q, and r $(p \vee r) \Rightarrow (q \wedge r)$ is true.

2. [20] Using sentential calculus (with a four column format), prove that the conclusion $s \wedge \sim s$ follows from premises: $p \Rightarrow q, \sim(q \vee r)$, and $r \Leftrightarrow \sim p$.

3. [20] Prove that the conclusion $q \wedge r$ follows from the premises $p \Rightarrow q, q \vee r, p \Rightarrow r, q \Rightarrow p$, and $r \Rightarrow p$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

4. [10] Using the predicates defined on the set □ of integers :

Px	x is prime,
Ox	x is odd,
Exy	x equal to y
Lxy	x is less than y.

Express in the syntax of Predicate Calculus (you may use any integers as constants):

a. *All prime numbers are positive and the only even prime number is 2.*

b. *There is no prime number strictly between 317 and 337.*

5. [25] Prove that $(\forall x)Px \wedge (\forall x)Qx$ follows from $(\forall x)(Px \wedge Qx)$ (Rather than using the TC rule be specific about the sentential calculus rule.)

6. [10] Using induction, prove that for $r \neq 1$ and $n \geq 0$, $\sum_{k=0}^{n} r^k = \frac{r^{n+1}-1}{r-1}$.

7. [10] Consider the Fibonacci sequence: $f_0 = 1, f_1 = 1, f_k = f_{k-1} + f_{k-2}$, for $k \geq 2$. Using induction, prove that for $n \geq 0$, $\sum_{k=0}^{n} f_k^2 = f_n f_{n+1}$.

8. [10] Prove for any sets A, B, C, and D that $(A \cup B) \sim (C \cup D) \subseteq (A \sim C) \cup (B \sim D)$.

9. [10]. Given sets $A, B,$ and C ,.prove that $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$.

10. [15]. Let R be defined $R = \{(x, y) : y \text{ is a multiple of } x\}$. Prove that R is a partial order on $\Box^{+}$.

11. [10] Let A be any set and R be a relation on A. Prove that if R is both symmetric and anti-symmetric then must also be transitive.

12. [10] Given $f : \Box^{+} \to \Box^{+}$ and $g : \Box^{+} \to \Box^{+}$ defined by $f(n) = n!$ and $g(n) = n - 1$, respectively, what are

a. $f \circ f$

b. $f \circ g$

c. $g \circ f$

d. $g \circ g$

13. Given a sets $A, B,$ and C and functions $f : A \to B$ and $g : B \to C$,

a. [10] Prove that if $g \circ f$ is one-to-one then f is one-to-one.

b. [10] Prove that if $g \circ f$ is onto then g is onto.