Practice Final Examination

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the $\boxed{\textbf{comments in boxes}}$

1. [10] Use a truth table to determine for which truth values of $p, q,$ and r $\sim(p \wedge r) \vee (\sim q \wedge r)$ is true.

2. [20] Using sentential calculus (with a four column format), prove that the conclusion p follows from premises: $p \vee q, q \Rightarrow t, \sim r \vee \sim s, (s \wedge t) \Rightarrow r,$ and $q \Rightarrow s$.

3. [20] Prove that the conclusion p follows from the premises $((p \Rightarrow q) \wedge (p \wedge \sim q)) \vee r$ and $r \Rightarrow p$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

4. [10] Using the predicates defined on the set L of upper case Latin characters, the set $\mathbb{Z}^+$ of positive integers, and the set S of strings of upper case Latin characters :

Vx	x is a vowel, for $x \in L$,
Sxn	x can be written in n strokes, for $x \in L$ and $n \in \mathbb{Z}^+$,
Wxs	x occurs in the string s, for $x \in L$ and $s \in S$,
Bxy	x occurs before y in the English alphabet, for $x, y \in L$.
Exy	x equals y, for $x, y \in L$.

Express in the syntax of Predicate Calculus (you may use upper case Latin characters, positive integers, and strings of upper case Latin characters as constants):

a. *'A' is the only upper case Latin character that is a vowel and can be written in three strokes but does not occur in the string 'STUPID'.*

b. *There is an upper case Latin character strictly between 'K' and 'R' that can be written in one stroke.*

5. [25] Prove that $(\forall u)(\exists v)Rvu$ follows from $(\exists x)(\forall y)Rxy$ (Rather than using the TC rule be specific about the sentential calculus rule.)

6. [10] Using induction, prove that for $n \geq 1$, $\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1$.

7. [10] a. Given a sequence of integers $a_1, a_2, \ldots$ such that $a_k > a_{k-1}$ for $k \geq 2$, using induction to prove that for $k \geq 1$, $a_k \geq a_1 + k - 1$. (Notice $a_k > a_{k-1}$ is equivalent to $a_k \geq a_{k-1} + 1$.)

b. [5] Using this, prove that for any integer m, display a k so that $a_k \geq m$.

8. [10] Prove for any sets $A, B,$ and C that $A \sim (B \sim C) = (A \sim B) \cup (A \cap C)$.

9. [10]. Given sets $A, B, C,$ and D be sets. Prove that $A \times B \subseteq C \times D$ if and only if $A \subseteq C \wedge B \subseteq D$.

10. **[15]**. Let R be defined $R = \{((x, y), (u, v)) : x^2 + y^2 = u^2 + v^2\}$. Prove that R is an equivalence relation on $\mathbb{R}^2$.

11.. Consider a relation R on a set A. Prove or disprove with a simple counter example each of the following:

a. [10] If R is reflexive, then R^2 is reflexive.

b. [10] If R is symmetric, then R^2 is symmetric.

c. [10] If R is antisymmetric, then R^2 is antisymmetric.

d. [10] If R is transitive, then R^2 is transitive.

12. [10] Given $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ and $g : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined by $f(n) = n^2$ and $g(n) = 2^n$, respectively, what are

a. $f \circ f$

b. $f \circ g$

c. $g \circ f$

d. $g \circ g$

13. [20] Given a sets A and B and function $f : A \to B$, Prove that f is one-to-one if and only if $f(X \cap Y) = f(X) \cap f(Y)$ for all $X, Y \subseteq A$. (Hint: Recall $f(\varnothing) = \varnothing$ and $f(\{x\}) = \{f(x)\}$ for any $x \in A$.)