Final Examination

CS 313H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the $\boxed{\textbf{comments in boxes}}$

1. [5] Use a truth table to determine for which truth values of $p, q,$ and r $\sim r \wedge (q \Leftrightarrow (p \vee \sim r))$ is true.

2. [15] Using the rules of Sentential Calculus conclude $(p \wedge \sim b) \Rightarrow c$ from the premises $a \vee (b \vee c)$ and $p \Rightarrow \sim a$.

3. [15] Prove that the conclusion p follows from premises: $p \vee q, q \Rightarrow t, \sim r \vee \sim s, (s \wedge t) \Rightarrow r,$ and $q \Rightarrow s$. First convert the premises and the negation of the conclusion into Conjunctive Normal Form, and then employ a resolution proof to get a contradiction.

4. Using the predicates defined on the set $\mathbf{P}$ of people:

Lxy $\quad x$ likes y ,
Exy $\quad x$ equal to y
Gxy $\quad x$ goes with y ,
And the constants Alice, Bill, and Clyde,

Express in the syntax of Predicate Calculus:

a. [5] *If anyone likes Alice it must be either Bill or Clyde.*

b. [5] *Bill is the only one who will go with Clyde but no one will go with Alice.*

c. [5] *A person goes with another person only if the first one likes the second*

5. [15] Prove that $(\forall x)(Px \wedge Rx) \wedge (\exists y)Qy$ follows from $(\forall x)(Px \wedge (Qx \wedge Rx))$.

6. [10] Using induction, prove that for $n \geq 0$, $3^{2n} + 4^{n+1}$ is an integer multiple of 5. (Hint: 9=5+4.)

7. Prove or give a simple counterexample:

a. [5] For any sets $A, B,$ and C that $(A \sim B) \cap C = (A \cap C) \sim B$..

b. [5] For any sets $A, B,$ and C that $A \sim (B \cup C) = (A \sim B) \cup (A \sim C)$..

8. [5]. Given sets $A, B, C,$ and D be sets. Prove that $A \times B \subseteq C \times D$ if $A \subseteq C \wedge B \subseteq D$.

9. [10} Prove that composition of relations is associative. That is if $R \subseteq A \times B, S \subseteq B \times C,$ and $T \subseteq C \times D,$ then $T \circ (S \circ R) = (T \circ S) \circ R.$

10. [10] Let E be a relation on $\mathbb{R}^3$ (Cartesian three space) defined by $((x, y, z), (u, v, w)) \in E$ if and only if $x^2 + y^2 + z^2 = u^2 + v^2 + w^2$. Prove that E is an equivalence relation.

11. [10] Given sets A and B and functions $f : A \to B$, and $g : B \to A$. Prove that f and g are both one-to-one and onto then $g \circ f$ is one-to-one and onto.

12. [10] Consider the parity function on integers: $p : \mathbb{Z} \to \{0,1\}$ where $p(n) = \begin{cases} 0 & n\, is\, even \\ 1 & n\, is\, odd \end{cases}$ and extend it to triples of integers by $p_3 : \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \to \{0,1\} \times \{0,1\} \times \{0,1\}$ where $p_3(m, n, r) = (p(m), p(n), p(r))$. Show that given any nine points in $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$, two must have the same parity.