1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes

1. [5] Given sets $A$ and $B$, each of cardinality $n \geq 1$, how many functions map $A$ in a one-to-one fashion onto $B$?

2 a. [5] Given the set of $r$ symbols $\{a_1,a_2,...,a_r\}$, how many different strings of length $n \geq 1$ exist?

b. [10] Given the set of $r$ symbols $\{a_1,a_2,...,a_r\}$, how many different strings of length $n \geq 2$ exist that contain at least one $a_1$ and at least one $a_2$? (Assume $r \geq 2$.)

3. [10] Present a combinatorial argument that for all positive integers $n$:

$$3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k.$$

b. [10] Present a combinatorial argument that for all integers $n \geq 3$:

$$\binom{3n}{3} = 3 \binom{n}{3} + 3 \cdot 2n \binom{n}{2} + n^3$$

(Hint: Consider three pairwise disjoint sets of cardinality $n$.)

4. [10] A multiset is similar to a set in that order is irrelevant but multiple copies of elements are allowed. For example, the sets $\{1,2,3\}$ and $\{1,1,1,2,2,3\}$ are identical and each has cardinality three but the multisets $\{1,2,3\}$ and $\{1,1,1,2,2,3\}$ are different and the first has cardinality three but the second has cardinality six. How many multisets of cardinality $n$ are there that employ elements from $\{1,1,1,2,2,3\}$?

5 a. [5] a. How many strings are there of length $k \geq 1$ using elements from the set $\{1,2,...,n\}$? (Repetition is allowed.)

b. [10] Now assume each of the different strings in part a. is equally likely. What is the probability that the minimum of the $k$ numbers is less than or equal to $r$, where $1 \leq r \leq n$?

6 a. [5] a. How many permutations of $a, b, c, d$, and $e$ have both $a$ to the left of $b$ and $b$ to the left of $e$?

b. [10] Assume all such permutations are equally likely, what is the probability that the permutation begins with $a$ given that it has both $a$ to the left of $b$ and $b$ to the left of $e$?