Examination 2

CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes. Unless commented, it will be assumed that you believe your solution is correct.

1. [20] Using only Definition 2', prove that the set of finitely long strings using characters from \{A, B, C, \ldots, Z, a, b, c, \ldots, z\} is infinite.

2. [20] Prove the set of intervals \([a,b] \mid 0 \leq a \leq b \leq 1\) is uncountably infinite.

3. [20] Let \(FP = \{\text{permutations of } \{0,\ldots,n\} \mid n \in \mathbb{N}\}\). Prove that \(FP\) is countably infinite.

4. [20] Prove that \(\sqrt{n^3} + 1 = o(n^7)\).

5. [20] Prove that for \(k \geq 1\), if for \(i = 1, 2, \ldots, k\), \(f_i = O(g)\), then \(\prod_{i=1}^{k} f_i = O(g^k)\).

6. [20] Prove that polynomials are asymptotically dominated by their largest power: That is, for \(k \geq 0\), \(\sum_{i=0}^{k} a_i n^i = O(n^k)\).