Examination 2

CS 336H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes. Unless commented, it will be assumed that you believe your solution is correct.

1. [20] Using only Definition 2', prove that the set of finitely long bit strings is infinite**.**

2. [20] Suppose operating system **S** allows passwords of 6 or more characters from the set $\{A,...,Z,a,...z,0,...,9,\_\}$ and no others. Is the set of legal passwords finite, countably infinite, or uncountably infinite? Prove your claim.

3. [20] Is the set of circles in the plane finite, countably infinite, or uncountably infinite**?** Prove your claim.

4. [20] Using no other asymptotic dominance theory than definitions, prove that $6n^{7/8} + 5n^{3/2} = \mathrm{O}(n^2)$.

5. [20] Employing induction prove that for $k \geq 1$, if for $i = 1, 2, \ldots, k$, $f_i = \mathrm{O}(f_{i+1})$, then $f_1 = \mathrm{O}(f_{k+1})$.

6. [20] Prove that $2^n = o(n!)$. (Hint: $\prod_{i=1}^{n} \frac{2}{i} = \prod_{i=1}^{3} \frac{2}{i} \cdot \prod_{i=4}^{n} \frac{2}{i} = \frac{4}{3} \prod_{i=4}^{n} \frac{2}{i}$ and $\frac{2}{i} \leq \frac{1}{2}$ for $i \geq 4$.)