

Name \_\_\_\_\_  
Row \_\_\_\_\_

**Homework 5**  
**CS 336**

**The important issue is the logic you used to arrive at your answer.**

1. How many 10 character words can be formed using exactly four  $a$ 's, three  $b$ 's, two  $c$ 's, and one  $d$ ?

2. Use a combinatorial argument to prove:  $\sum_{k=0}^n \binom{n}{k} r^k = (r+1)^n$ . (Hint: Consider a set of  $r+1$  elements as  $A \cup \{b\}$ , where  $A$  has  $r$  elements and  $b \notin A$ . Determine a situation that has  $(r+1)^n$  options and then count it another way to get  $\sum_{k=0}^n \binom{n}{k} r^k$ .)

3. Suppose all sequences of length  $n$  drawn from  $r$  distinct elements are equally likely. Assume that the elements can be strictly ordered:  $a_1 < a_2 < \dots < a_r$ . What is the probability that the sequence is non-decreasing? (From the use of “non-decreasing” rather than “increasing” you should assume that repetition is allowed.)