Homework 8 Solution to Problem 3 CS 336

3. a. For $k \ge 1$, assume all strings of length k from the set $\{X, Y, Z\}$ (allowing repetition) are equally likely. What is the probability that such a string has no Z?

There are 3^k equally likely strings of length k from the set $\{X, Y, Z\}$ (allowing repetition). Of these 2^k have no Z. The probability that such a string has no Z is then $\left(\frac{2}{3}\right)^k$.

b. What is the probability that such a string has no Y given that it has no Z?

Only one string has no Y and no Z. Thus the probability of that string is $\left(\frac{1}{3}\right)^k$ and the probability of such a string given that it has no Z is then $\left(\frac{1}{3}\right)^k / \left(\frac{2}{3}\right)^k = \left(\frac{1}{2}\right)^k$.