

**Homework 14**  
**CS 336**

Name \_\_\_\_\_  
Seating Section: **1 2 3 4 5 6**

**The important issue is the logic you used to arrive at your answer.**

1. Consider the functions  $f$  and  $g$  defined on  $\mathbf{N}$  by  $f(n) = \begin{cases} n^2 & \text{for } n \text{ even} \\ 2n & \text{for } n \text{ odd} \end{cases}$  and  $g(n) = n^2$ . Show that  $f = O(g)$  and  $g \neq O(f)$ .

2. Using Theorem 2 and induction, prove that if for  $i = 1, 2, \dots, k$ ,  $f_i = O(g_i)$ , then

$$\sum_{i=1}^k f_i = O\left(\sum_{i=1}^k |g_i|\right).$$

3. Construct a simple counter-example such that  $f_1 = \mathbf{O}(g_1)$  and  $f_2 = \mathbf{O}(g_2)$  but  $f_1 + f_2 \neq \mathbf{O}(g_1 + g_2)$ .