Name ______________________________

Homework 23 Seating Section: **R M L**

CS 336

The important issue is the logic you used to arrive at your answer.

1. Let A be the set of finite length strings of 0s and 1s and let $\boldsymbol{s}$ = <010101...> (i.e., the infinite string alternating 0s and 1s). Let B be the set of all infinite strings of the form $\boldsymbol{a}||\boldsymbol{s}$ where $\boldsymbol{a} \in A$ and || indicates concatenation. Is B finite, countably infinite, or uncountably infinite? Prove your assertion.

2. Using only Definition 2', show that the unit circle $A = \{(x,y) | x \text{ and } y \text{ are real and } x^2 + y^2 \leq 1\}$ is infinite.

3. Consider arrays of positive integers whose sum is 17 (e.g., <17>, <9, 8>, and <1, 5, 1, 6, 4>). Is the set of all such arrays finite, countably infinite, or uncountably infinite? Prove your assertion.

4. Since there are $n!$ permutations of n elements, a binary decision tree whose leaves are the $n!$ permutations must have height $\lceil \log_2 n! \rceil$. This is thus the number of comparisons necessary to determine which permutation one has - thus to sort the elements. Yet this number is often quoted as $n \log_2 n$. Prove that $\log_2 n! = \mathrm{O}(n \log_2 n)$.

5. Show with a simple counter-example that even if $f = \mathrm{O}(g)$ and $a > 1$, it does not follow that $a^f = \mathrm{O}(a^g)$. (Hint: you may want to use that if $0 < c < b$, then $b^n \neq \mathrm{O}(c^n)$.)

6. Suppose $f_1 = \mathrm{O}(g_1)$ and $f_2 = o(g_2)$, show $f_1 + f_2 = \mathrm{O}(|g_1| + |g_2|)$.