1. [5] How many strings of 0s and 1s of length 8 have an even number of 0s or an even number of 1s or both?

2 a. [5] For \( n \geq 2 \), how many permutations of \( a_1,a_2,\ldots,a_n \) have \( a_1 \) in positions 1, 2, or 3?

b. [5] For \( n \geq 2 \), how many permutations of \( a_1,a_2,\ldots,a_n \) have \( a_1 \) in positions 1, 2, or 3 and \( a_2 \) in positions 2, 3, or 4?

3.a [10] Present a combinatorial argument that for all positive integers \( n \)

\[
\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}.
\]

(Hint: Consider distinct sets A and B each of cardinalities \( n \).)

b [10] Present a combinatorial argument that for all positive integers \( 1 \leq k < m \):

\[
m! = \binom{m}{k} k! (m-k)!.
\]

4. [10] Consider 5-tuples of the form \(< r_1,r_2,r_3,r_4,r_5 >\), where the \( r_i \geq 0 \). How many such 5-tuples are there satisfying

\[
r_1 + r_2 + r_3 + r_4 + r_5 \leq 20.
\]

5. [10] For \( n \geq 5 \), consider strings of length \( n \) using upper case roman letters (i.e., \{ \( A, B, C,\ldots,X,Y,Z \} \)). Assuming all such strings are equally likely what is the probability that the string occurs in non-decreasing order (i.e. all \( A \)s precede all \( B \)s, all \( B \)s precede all \( C \)s ,… etc.)?

6. [10] Consider strings of length \( n \geq 2 \) containing exactly \( k \) 1’s and \( n-k \) 0’s and having no adjacent 1’s (i.e., there is at least one 0 between any 1’s). Assuming \( k \geq 1 \) and \( n \geq 2k \), how many such strings are there? (Hint: Consider cases based upon the contents of the last position.)