Practice Examination 1

CS 336

1. [5] How many strings of *0*s and *1*s of length 8 have an even number of *0*s or an even number of *1*s or both?

2 a. [5] For $n \geq 2$, how many permutations of $a_1, a_2, ..., a_n$ have a_1 in positions 1, 2, or 3?

[5] b. For $n \geq 2$, how many permutations of $a_1, a_2, ..., a_n$ have a_1 in positions 1, 2, or 3 and a_2 in positions 2, 3, or 4?

3.a [10] Present a combinatorial argument that for all positive integers *n*

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}.$$

(Hint: Consider distinct sets A and B each of cardinalities *n*.)

b [10] Present a combinatorial argument that for all positive integers $1 \leq k < m$:

$$m! = \binom{m}{k} k!(m-k)!.$$

4. [10] Consider 5-tuples of the form $< r_1, r_2, r_3, r_4, r_5 >$, where the $r_i \geq 0$. How many such 5-tuples are there satisfying

$$r_1 + r_2 + r_3 + r_4 + r_5 \leq 20?$$

5. [10] For $n \geq 5$, consider strings of length n using upper case roman letters (i.e., {*A, B, C,…,X,Y,Z*}). Assuming all such strings are equally likely what is the probability that the string occurs in non-decreasing order (i.e. all *A*s precede all *B*s, all *B*s precede all *C*s ,… etc.)?

6. [10] Consider strings of length $n \geq 2$ containing exactly k 1's and $n-k$ 0's and having no adjacent 1's (i.e., there is at least one 0 between any 1's). Assuming $k \geq 1$ and $n \geq 2k$, how many such strings are there? (Hint: Consider cases based upon the contents of the last position.)