| | | |
|---|---|---|
| 1 | 5 | |
| 2 | 10+10 | |
| 3 | 5+10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10+5 | |
| Total | 75 | |

Name ______________________________

**Examination 1**

**CS 336H**

**1. The important issue is the logic you used to arrive at your answer.**
**2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.**
**3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.**
**4. Comment on all logical flaws and omissions and enclose the** **comments in boxes**

**1. [5]** Given $m$ 1's, $n$ 2's, and $p$ 3's, how many distinct sequences are there that employ each of the $m+n+p$ symbols? (Note: All 1's are identical as are all 2's and all 3's)

**2.a [10]** Present a combinatorial argument that for all positive integers $x$ and $y$

$$\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n.$$

(Hint: Consider sequences drawn from the union of distinct sets A and B of cardinalities $x$ and $y$, respectively.)

**b [10]** Present a combinatorial a combinatorial argument that for all positive integers $1 \le k \le m \le r$:

$$\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}.$$

**3 a. [5]** How many permutations of *a*, *b*, *c*, *d*, *e*, and *f* have *a* to the left of *b* and *b* to the left of *c*?

**b. [10]** How many permutations of *a*, *b*, *c*, *d*, *e*, and *f* have *a* to the left of *b* **or** *b* to the left of *c*?

**4. [10]** Consider 5-tuples of the form $\langle r_1, r_2, r_3, r_4, r_5 \rangle$, where the $r_i \ge 1$. How many such 5-tuples are there satisfying

$$r_1 + r_2 + r_3 + r_4 + r_5 = 20?$$

**(Hint: You have seen the problem with the restriction** $r_i \ge 0$. Can you make a small change to guarantee $r_i \ge 1$?)

**5. [10]** Consider strings of length $n \ge 2$ containing exactly $k$ 1's and $n-k$ 0's and having no adjacent 1's (i.e., there is at least one 0 between any 1's). Assuming $k \ge 1$ and $n \ge 2k$, how many such strings are there?

**6. a. [10]** For $n \geq 6$, consider strings of length $n$ using elements of $\{a, b, c, d, e\}$. Assume all such strings are equally likely. What is the probability that a string has three or more $a$'s?

**b. [5]** What is the probability that such a string has exactly three $b$'s given that it has exactly three $a$'s?