1	5	
2	10	
3	20	
4	10	
5	15	
Total	60	

Name ______________________________

Examination 1

CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [5] Given $n \geq 9$, how many strings of length n containing *0*s, *1*s, *2*s, and *3*s, have exactly three *0*s, exactly four *1*s, and either one or two *2*s (and the rest *3*s)?

2. For $n \geq 1$, let A and B be disjoint sets, each of cardinality n, and $C = A \cup B$. Consider functions $f: C \to C$.

[5] a. How many such functions are there that map A to B and B to A (i.e., for all $x \in C$, if $x \in A$ then $f(x) \in B$ and if $x \in B$ then $f(x) \in A$)?

[5] b. How many such **one-to-one** functions are there that map A to B and B to A?

3.a [10] Present a combinatorial argument that for all positive integers $n, p,$ and q:

$$\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = (p+q)^n .$$

b [10] Present a combinatorial argument that for all integers p and n so that $1 \leq p \leq n$:

$$\binom{n+3}{p} = \binom{n}{p} + \binom{3}{1}\binom{n}{p-1} + \binom{3}{2}\binom{n}{p-2} + \binom{n}{p-3}.$$

4. [10] Given positive integers p and n, in how many ways can p identical tokens be distributed to n different people so that no person has all of the tokens?

5. Consider **six** card hands drawn for a 52 card deck and assume all such are equally likely.

a.[5] What is the probability that the hand has exactly five clubs?

b.[10] What is the probability that the hand has exactly five clubs **given that it has at least one spade**?