

1	5	
2	10	
3	20	
4	10	
5	15	
Total	60	

Examination 1

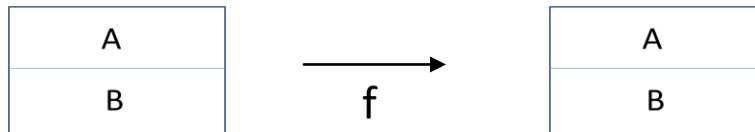
Name _____

CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes

1. [5] Given $n \geq 9$, how many strings of length n containing 0s, 1s, 2s, and 3s, have exactly three 0s, exactly four 1s, and either one or two 2s (and the rest 3s)?

2. For $n \geq 1$, let A and B be disjoint sets, each of cardinality n , and $C = A \cup B$. Consider functions $f: C \rightarrow C$.



[5] a. How many such functions are there that map A to B and B to A (i.e., for all $x \in C$, if $x \in A$ then $f(x) \in B$ and if $x \in B$ then $f(x) \in A$)?

[5] b. How many such **one-to-one** functions are there that map A to B and B to A ?

3.a [10] Present a combinatorial argument that for all positive integers n, p , and q :

$$\sum_{k=0}^n \binom{n}{k} p^k q^{n-k} = (p+q)^n.$$

b [10] Present a combinatorial argument that for all integers p and n so that $1 \leq p \leq n$:

$$\binom{n+3}{p} = \binom{n}{p} + \binom{3}{1} \binom{n}{p-1} + \binom{3}{2} \binom{n}{p-2} + \binom{n}{p-3}.$$

4. [10] Given positive integers p and n , in how many ways can p identical tokens be distributed to n different people so that no person has all of the tokens?

5. Consider **six** card hands drawn from a 52 card deck and assume all such hands are equally likely.

a.[5] What is the probability that the hand has exactly five clubs?

a.[10] What is the probability that the hand has exactly five clubs **given that it has at least one spade**?