Examination 2

CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [20] Using only Definition 2', show that the set positive, integral multiples of three (i.e., $\{3, 6, 9, \ldots\} = \{3k \mid k \in \mathrm{N} \text{ and } k \geq 1\}$) is infinite**.**

2. [20] Consider the set $A = \{(p,q,r) \mid p,q,r \in \mathrm{N}\}$ (i.e., ordered triples of natural numbers). Is A finite, countably infinite, or uncountably infinite? Prove your claim.

3. [20] 3. [20] Show that the set of points in the square $B = \{(x, y) \mid -1/2 \leq x \leq 1/2 \text{ and } 1 \leq y \leq 2\}$ is uncountably infinite.

4. [20] Using only Definition 1, prove that for any $a \geq 0$, $n^4 = \mathrm{O}(n^{4.01})$.

5. **[20]** Given $f = \mathrm{O}(g)$ and $g = o(h)$, either proof that $f = o(h)$ or construct a simple counterexample to prove that f is not necessarily $o(h)$.

6. **[20]** Prove that for $0 < b < a$, $a^n \neq \mathrm{O}(b^n)$.