

## Examination 2

### CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes. Unless commented, it will be assumed that you believe your solution is correct.

1. [20] Using only Definition 2', prove that the set of infinite strings of 0s and 1s is infinite.

2. [20] Prove the set of intervals  $\{[a, b] \mid 0 \leq a \leq b \leq 1\}$  is uncountably infinite.

3. [20] Let  $FP = \{\text{permutations of } \{0, \dots, n\} \mid n \in \mathbb{N}\}$ . Prove that  $FP$  is countably infinite.

4. [20] a. By induction prove that  $n \geq 1, n^{n-1} \geq n!$ .

b. Using part a, prove that  $n^n \neq O(n!)$ . (You may ignore part a if you have another way of proving this and you may use part a even if you weren't able to prove it above.)

5. [20] Prove that  $2^n = o(n!)$ . (Hint:  $\prod_{i=1}^n 2 = \prod_{i=1}^n \frac{2}{i} i$ )

6. [20] Prove that for  $k \geq 0, \sum_{i=0}^k a_i n^i = O(n^k)$ .