How to do well on the Cardinality/Asymptotic Dominance Exam

The numbering of the theorems here corresponds to the numbering on the exam theory handout and may differ from the theory handouts from the past.

1. Definition 2' Problems: Make sure these items are presented absolutely clearly:

   a. A function from the set to the set is presented. (Being clear about the function is not to be confused with comments about how you constructed the function. If you have information about how you constructed the function, place it on a separate piece of paper and throw it away when turning in your exam.)

   b. That function is shown to be one-to-one.

   c. That function is shown not to be onto (i.e. some element of the set is mapped to by no element).

2. Proving a set is countably infinite using definition 3: Make sure these items are presented absolutely clearly:

   a. A function from the natural number into the set is presented. (See above about clarity and the irrelevance of showing how the function was constructed.)

   b. That function is shown to be one-to-one.

   c. That function is shown to be onto.

3. Proving a set is countably infinite using definition 3: Make sure these items are presented absolutely clearly:

   a. A function from the natural number into the set is presented. (See above about clarity and the irrelevance of showing how the function was constructed.)

   b. That function is shown to be one-to-one.

   c. That function is shown to be onto.

4. Proving a set is countably infinite using Theorem 7: Make sure these items are presented absolutely clearly:

   a. A function from the natural number into the set is presented. (See above about clarity and the irrelevance of showing how the function was constructed.)

   b. That function is shown to be onto.

   c. The set is infinite (using definition 2' or Theorems 1, 2, 3, and/or 4).

5. Proving a set is countably infinite using Theorem 10: Make sure these items are presented absolutely clearly:

a. Display a set of finite sets whose union forms the original set.

b. The set is infinite (using definition 2' or Theorems 1, 2, 3, and/or 4).

6. Proving that a set is uncountably infinite using Theorem 12: Make sure these items are presented absolutely clearly:

a. A function from a known uncountably infinite set (see Theorems 5 and 6) into the set is presented. (See above about clarity and the irrelevance of showing how the function was constructed.)

b. That function is shown to be one-to-one.

7. Showing $f = O(g)$: The proof almost certainly will begin with "Take $M = ..., N = ...,$ then for $n \geq N$" and then have a line of inequalities of the form $|f(n)| \leq ... \leq M|g(n)|$. (Be careful about absolute values and make sure you specify *M* and *N* but do not submit any description of how you determined them.)

8. Showing $f \neq O(g)$: The proof almost certainly will begin with "For all M and N, take $n = expression\, in\, M\ and\ /\ or\ N$ and then have a line of inequalities of the form $|f(n)| \geq ... \geq M|g(n)|$ with at least one strict inequality. (Be careful about absolute values and make sure you clearly define *n*. Do not submit any description of how you determined the expression for *n* .)

9. Showing $f = o(g)$: The proof almost certainly will begin with "For any $\varepsilon > 0$, take $N = expression,$ then for $n \geq N$" and then have a line of inequalities of the form $|f(n)| \leq ... \leq \varepsilon |g(n)|$. (Be careful about absolute values and make sure you specify *N* but do not submit any description of how you determined it.)

10. Showing $f \neq o(g)$: The proof almost certainly will begin with "Take $\varepsilon = expression$, for any N, take $n = expression,$ then $n \geq N$ and ... " and then have a line of inequalities of the form $|f(n)| \geq ... \geq \varepsilon |g(n)|$ with at least one strict inequality. (Be careful about absolute values and make sure you specify ε and the expression for *n* but do not submit any description of how you determined that expression or ε .)