

## Fibonacci Code Verification

Given the Fibonacci sequence,  $f_0 = 0, f_1 = 1$ , and  $f_i = f_{i-1} + f_{i-2}$ , prove that the program segment

```

fibm2 := 0
fibm1 := 1
i := 2
while i ≤ n do
  fib := fibm1 + fibm2
  fibm2 = fibm1
  fibm1 = fib
  i := i + 1
end

```

is partially correct with respect to precondition " $n \geq 2$ " and postcondition " $fib = f_n$ ".

$$\begin{array}{l}
 \text{_____ } n \geq 2 \\
 \text{fibm2 := 0 } \text{_____ } n \geq 2 \wedge \text{fibm2} = 0 \\
 \text{fibm1 := 1 } \text{_____ } n \geq 2 \wedge \text{fibm2} = 0 \wedge \text{fibm1} = 1 \\
 \text{i := 2 } \text{_____ } n \geq 2 \wedge \text{fibm2} = 0 \wedge \text{fibm1} = 1 \wedge i = 2 \\
 \text{_____ } i \leq n + 1 \wedge \text{fibm2} = f_{i-2} \wedge \text{fibm1} = f_{i-1} \wedge (i \geq 3 \Rightarrow \text{fib} = f_{i-1}) \wedge n \geq 2 \\
 \text{while } i \leq n \text{ do } \text{_____ } i \leq n \wedge \text{fibm2} = f_{i-2} \wedge \text{fibm1} = f_{i-1} \wedge (i \geq 3 \Rightarrow \text{fib} = f_{i-1}) \wedge n \geq 2 \\
 \quad \text{fib := fibm1 + fibm2 } \text{_____ } i \leq n \wedge \text{fibm2} = f_{i-2} \wedge \text{fibm1} = f_{i-1} \wedge \text{fib} = f_i \wedge n \geq 2 \\
 \quad \text{fibm2 = fibm1 } \text{_____ } i \leq n \wedge \text{fibm2} = f_{i-1} \wedge \text{fibm1} = f_{i-1} \wedge \text{fib} = f_i \wedge n \geq 2 \\
 \quad \text{fibm1 = fib } \text{_____ } i \leq n \wedge \text{fibm2} = f_{i-1} \wedge \text{fibm1} = f_i \wedge \text{fib} = f_i \wedge n \geq 2 \\
 \quad \text{i := i + 1 } \text{_____ } i \leq n + 1 \wedge \text{fibm2} = f_{i-2} \wedge \text{fibm1} = f_{i-1} \wedge \text{fib} = f_{i-1} \wedge n \geq 2 \\
 \text{end } \text{_____ } i \leq n + 1 \wedge \text{fibm2} = f_{i-2} \wedge \text{fibm1} = f_{i-1} \wedge (i \geq 3 \Rightarrow \text{fib} = f_{i-1}) \wedge n \geq 2 \\
 \text{_____ } i > n \wedge i \leq n + 1 \wedge \text{fibm2} = f_{i-2} \wedge \text{fibm1} = f_{i-1} \wedge (i \geq 3 \Rightarrow \text{fib} = f_{i-1}) \wedge n \geq 2 \\
 \text{_____ } i = n + 1 \wedge (i \geq 3 \Rightarrow \text{fib} = f_{i-1}) \wedge n \geq 2 \\
 \text{_____ } \text{fib} = f_n
 \end{array}$$