

9. [10] Prove correct with respect to precondition that a , b , and c are defined and postcondition $(\max = a \vee \max = b \vee \max = c) \wedge (\max \geq a) \wedge (\max \geq b) \wedge (\max \geq c)$:

```

max := a
if b > a then
  {if b > c then
    max := b
  else
    max := c}
else
  {if c > a then
    max := c}
  
```

max := a	max = a
if b > a then	$(\max = a) \wedge (b > a)$
{ if b > c then	$(\max = a) \wedge (b > a) \wedge (b > c)$
max := b	$(\max = b) \wedge (b > a) \wedge (b > c)$
else	$(\max = a) \wedge (b > a) \wedge (b \leq c)$
max := c}	$(\max = c) \wedge (b > a) \wedge (b \leq c)$
_____	$((\max = b) \wedge (b > a) \wedge (b > c)) \vee ((\max = c) \wedge (b > a) \wedge (b \leq c))$
else	$(\max = a) \wedge (b \leq a)$
{ if c > a then	$(\max = a) \wedge (b \leq a) \wedge (c > a)$
max := c}	$(\max = c) \wedge (b \leq a) \wedge (c > a)$
_____	$((\max = c) \wedge (b \leq a) \wedge (c > a)) \vee ((\max = a) \wedge (b \leq a) \wedge (c \leq a))$
_____	$((\max = c) \wedge (b \leq a) \wedge (c > a)) \vee ((\max = a) \wedge (b \leq a) \wedge (c \leq a))$
_____	$\vee ((\max = b) \wedge (b > a) \wedge (b > c)) \vee ((\max = c) \wedge (b > a) \wedge (b \leq c))$
_____	$((\max = c) \wedge (c > a \geq b)) \vee ((\max = a) \wedge (b \leq a) \wedge (c \leq a))$
_____	$\vee ((\max = b) \wedge (b > a) \wedge (b > c)) \vee ((\max = c) \wedge (c \geq b > a))$
_____	$(\max = a \vee \max = b \vee \max = c) \wedge (\max \geq a) \wedge (\max \geq b) \wedge (\max \geq c)$

$$\begin{aligned}
wp(\mathbf{if} \ x > y \ \mathbf{then} \ x := y, x \leq y) &= ((x > y) \Rightarrow \text{true}) \wedge ((x \leq y) \Rightarrow (x \leq y)) \\
&= \text{true} \wedge \text{true} \\
&= \text{true}
\end{aligned}$$