Combinatorics and Probabilities for Poker Hands

Hand	Combinatorics	Number	Probability
Total	$\binom{52}{5}$	2,598,960	1.0000
One pair	$13\binom{4}{2}\binom{12}{3}4^3$	1,098,240	.4226
Two pair	$\binom{13}{2}\binom{4}{2}\binom{4}{2}44$	123,552	.0475
Three of a kind	$13\binom{4}{3}\binom{12}{2}4^2$	54,912	.0211
Straight	$10\cdot 4^5 - 10\cdot 4$	10,200	.0039
Flush	$4\binom{13}{5} - 10\cdot 4$	5,108	.0020
Full House	$13\binom{4}{3}12\binom{4}{2}$	3,744	.0014
Four of a kind	$13\cdot 48$	624	.0002
Straight flush	$10\cdot 4$	40	$1.5\cdot 10^{-5}$ 1

For each of the hands, we consider only that and nothing better. Thus, the hands with straights do not include those with straight flushes, the hands with pairs do not include those with two pairs or three of a kind, etc.

(The so-called "royal flush" is actually just a particular straight flush. There are four of them.)

Probabilities of Increasing Hands with a Draw

Original Hand	Intended Hand	Discards	Combinatorics	Numbers	Probability
One pair	Two pair	3	$\frac{(3\binom{3}{2}+9\binom{4}{2})\cdot 41}{\binom{47}{3}}$	$\frac{2,583}{16,215}$	.1593
One pair	Three of a kind	3	$\frac{2\binom{45}{2}-FH^{**}-FoaK^{**}}{\binom{47}{3}}$	$\frac{1,770}{16,215}$	.1092
One pair	Full House	3	$\frac{(3+9\binom{4}{3})+2\cdot(3\binom{3}{2}+9\binom{4}{2})}{\binom{47}{3}}$	$\frac{165}{16,215}$	.0102
One pair	Four of a kind	3	$\frac{45}{\binom{47}{3}}$	$\frac{45}{16,215}$	.0028
Two pair	Full House	1	$\frac{2\cdot 2}{47}$	$\frac{4}{47}$	.0851
Three of a kind	Full House	2	$\frac{2\binom{3}{2}+10\binom{4}{2}}{\binom{47}{2}}$	$\frac{66}{1081}$	.0611
Three of a kind	Four of a kind	2	$\frac{46}{\binom{47}{2}}$	$\frac{46}{1081}$	.0426
Four card straight (outside)	Straight*	1	$\frac{2\cdot 4}{47}$	$\frac{8}{47}$	.1702
Four card straight (inside)	Straight*	1	$\frac{4}{47}$	$\frac{4}{47}$	.0851
Four card flush	Flush*	1	$\frac{9}{47}$	$\frac{9}{47}$	.1915

* - includes the possibility of a straight flush.
** - where *FH* is 165 (the number of possible full houses) and *FoaK* is 45 (the number of possible fours of a kind)