For Thursday, Sept. 20, prove the following lemma:

Lemma: Given $a < b$, a function g with N continuous derivatives on $[a, b]$ and zeros on $[a, b]$ of total multiplicity N+1, then there exists a point $\xi \in [a,b]$ so that $g^{(N)}(\xi) = 0$.

(Remember a function g such that $g^{(j)}(x_i) = 0$, for $i = 1,\dots,n$ and $j = 0,\dots,k_i$, is said to have zeros of total multiplicity $\sum_{i=1}^{n}(k_i + 1)$.)

And then fill in the spaces and prove the theorem:

Theorem: Given $a < b$, a function h with ___ continuous derivatives on $[a, b]$, a polynomial f of degree ____ so that

$$f^{(j)}(x_i) = h^{(j)}(x_i) \text{ for } i = 1,\dots,n \text{ and } j = 0,\dots,k_i,$$

where the set $x_i \in [a,b]$ and are distinct, $k_i \geq 0$ for $i = 1,\dots,n$, and $N = \sum_{i=1}^{n}(k_i + 1)$ and then for $x \in [a,b]$ there exists a point $\xi \in [a,b]$ so that

$$h(x) - f(x) = \_\_\_\_\_\_\_\_$$