1	20	
2	15	
3	10	
4	10	
5	30	
6	20	
Total	105	

Name ______________________________

Examination 1
M340L-CS

1. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
2. Do not submit the scratch sheets. However, all of the work necessary to obtain the solution should be on these sheets.
3. Comment on all errors and omissions and enclose the **comments in boxes**

1. Mark T(rue) or F(alse) for each of the following statements:

__T__a. Every elementary row operation is reversible.

__F__b. Two matrices are row equivalent if they have the same number of rows.

__F__c. Elementary row operations on an augmented matrix can change the solution set of the associated linear system.

__T__d. A consistent system of linear equations always has one or more solutions.

__F__e. The reduced echelon form of a matrix may not be unique.

__T__f. A general solution of a system is an explicit description of all solutions of the system.

__T__g. If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $Ax = b$ is consistent for each b in $\mathbb{R}^m$.

__T__h. If the equation $Ax = b$ is consistent, then b is in the set spanned by the columns of A.

__F__i. For an $n \times n$ system of linear equations, the Gaussian Elimination Algorithm with Partial Pivoting and Elimination Separated from Solving uses approximately n^2 floating point multiplications and n^2 floating point additions/subtractions.

__F__j. The original Gaussian Algorithm can have multipliers no larger than one in absolute value.

2. Find the general solutions of the systems whose **augmented** matrix is:

$$\begin{bmatrix} 0 & 2 & -4 & 0 & 6 \\ 1 & -3 & 4 & 1 & -12 \\ 2 & 4 & 0 & 0 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 4 & 1 & -12 \\ 0 & 2 & -4 & 0 & 6 \\ 2 & 4 & 0 & 0 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 4 & 1 & -12 \\ 0 & 2 & -4 & 0 & 6 \\ 0 & 10 & -8 & -2 & 30 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -3 & 4 & 1 & -12 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 10 & -8 & -2 & 30 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 4 & 1 & -12 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 12 & -2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 1 & -3 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 12 & -2 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & -2 & 1 & -3 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 1 & -1/6 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 2/3 & -3 \\ 0 & 1 & 0 & -1/3 & 3 \\ 0 & 0 & 1 & -1/6 & 0 \end{bmatrix}$$

Thus x_4 is free, $x_3 = 0-(-1/6)x_4 = x_4/6, x_2 = 3-(-1/3)x_4 = 3+x_4/3,$ and $x_1 = -3-2/3x_4 = -3-2x_4/3.$

3. Show that if there are two distinct solutions to a linear system of equations then there is an infinite set of solutions. More specifically, show that if $Ax = b$ and $Ay = b$, then for any scalar α, $A(x+\alpha(y-x)) = b.$

If $Ax = b$ and $Ay = b$ then $A(y-x) = Ay - Ax = b - b = 0$ and for any scalar α, $A(x+\alpha(y-x)) = Ax + A\alpha(y-x) = Ax + \alpha A(y-x) = b + \alpha 0 = b.$

4. Suppose you are to solve *m* different linear systems of *n* equations in *n* unknowns. All of the equations have the same matrix, however, they just differ in right hand sides. Estimate how many ***multiplications*** are required.

The elimination step on the matrix requires $\frac{n^3}{3}$ multiplications and each of the *m* different linear systems requires n^2 multiplications for a total of $\frac{n^3}{3} + mn^2$ multiplications.

5. Use the Gaussian Elimination with Partial Pivoting and Solution algorithm to solve

$$3x_1 + 5x_2 - 2x_3 = -16$$
$$-3x_1 \qquad - x_3 = -5$$
$$6x_1 + 2x_2 + 4x_3 = 8$$

Show what occupies storage in the A matrix and the ip array initially and after each major step of elimination.

A | ip

				ip
3	5	-2		?
-3	0	-1		?
6	2	4		?

				ip
6	2	4		3
-1/2	1	1		?
1/2	4	-4		?

				ip
6	2	4		3
-1/2	4	-4		3
1/2	1/4	2		?

				ip
6	2	4		3
-1/2	4	-4		3
1/2	1/4	2		3

For the elimination applied to $b = \begin{bmatrix} -16 \\ -5 \\ 8 \end{bmatrix}$

we get it changing to $\begin{bmatrix} 8 \\ -5 \\ -16 \end{bmatrix}, \begin{bmatrix} 8 \\ -1 \\ -20 \end{bmatrix}, \begin{bmatrix} 8 \\ -20 \\ -1 \end{bmatrix},$

and finally $\begin{bmatrix} 8 \\ -20 \\ 4 \end{bmatrix}$. In the bottom loop we

$b_3 = 4$ and $x_3 = \frac{4}{2} = 2.$ Then we get

$b_2 = -20 - (-4) \cdot 2 = -12$ so $x_2 = \frac{-12}{4} = -3.$

Finally we get $b_1 = 8 - 2 \cdot (-3) - 4 \cdot 2 = 6$ so

$x_1 = \frac{6}{6} = 1.$

6. Fill in the five blanks in the code for Gaussian Elimination with Partial Pivoting and Solution separated from elimination:

for k = 1:n
 choose ip_k such that $|A_{ip_k,k}| = \max\{|A_{i,k}| : i \geq k\}$
  if $A_{ip_k,k}$ = 0
   warning ('Pivot in Gaussian Elimination is zero')
  end
  swap $A_{k,k},\ldots,A_{k,n}$ with $A_{ip_k,k},\ldots,A_{ip_k,n}$
  for i = $\underline{k+1}$:n
   $A_{i,k}$ = $\underline{A_{i,k} / A_{k,k}}$
   for j = k+1:n
    $A_{i,j} = \underline{A_{i,j} - A_{i,k} A_{k,j}}$
   end
  end
end
for k = 1:n
  swap b_k with $\underline{b_{ip_k}}$
  for i = k+1:n
   $b_i = b_i - A_{i,k} b_k$
  end
end
for i = n:-1:1
  for j = i+1:n
   $b_i = b_i - A_{i,j} x_j$
  end
  $x_i = \underline{b_i / A_{i,i}}$
end