1	20	
2	15	
3	10	
4	10	
5	30	
6	20	
Total	105	

Name ______________________________

Examination 1
M340L-CS

1. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
2. Do not submit the scratch sheets. However, all of the work necessary to obtain the solution should be on these sheets.
3. Comment on all errors and omissions and enclose the $\boxed{\textbf{comments in boxes}}$

1. [20] Mark T(rue) or F(alse) for each of the following statements:

____a. A consistent system of linear equations could have no solution.

____b. The column space of A is the set of all vectors that can be written as Ax for some x .

____c. The solution set of the linear system whose augmented matrix is $\begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{b} \end{bmatrix}$ is the same as the solution set of the equation $x_1\boldsymbol{a}_1 + x_2\boldsymbol{a}_2 + x_3\boldsymbol{a}_3 = \boldsymbol{b}$.

____d. When $\boldsymbol{u}$ and $\boldsymbol{v}$ are nonzero vectors, Span $\{\boldsymbol{u}, \boldsymbol{v}\}$ contains only the line through $\boldsymbol{u}$ and the origin, and the line through $\boldsymbol{v}$ and the origin.

____e. A vector $\boldsymbol{b}$ is a linear combination of the columns of a matrix $\boldsymbol{A}$ if and only if the equation $\boldsymbol{Ax} = \boldsymbol{b}$ has at least one solution.

____f. Any linear combination of vectors can always be written in the form $\boldsymbol{Ax}$ for a suitable matrix $\boldsymbol{A}$ and vector $\boldsymbol{x}$.

____g. The equation $\boldsymbol{x} = \boldsymbol{p} + t\boldsymbol{v}$ describes a line through $\boldsymbol{v}$ parallel to $\boldsymbol{p}$.

____h. The columns of a matrix $\boldsymbol{A}$ are linearly independent if the equation $\boldsymbol{Ax} = \boldsymbol{0}$ has the trivial solution.

____i. The columns of any 4×5 matrix are linearly dependent.

____j. If $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4, \text{and} \boldsymbol{v}_5$ are in $\mathbb{R}^5$ and $\boldsymbol{v}_3 = \boldsymbol{0}$, then $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4, \boldsymbol{v}_5\}$ is linearly dependent.

2. Find the general solution of the systems whose **augmented** matrix is:

$$\begin{bmatrix} 1 & -2 & -1 & 3 & 0 \\ -2 & 4 & 5 & -5 & 3 \\ 3 & -6 & -6 & 8 & -3 \end{bmatrix}$$

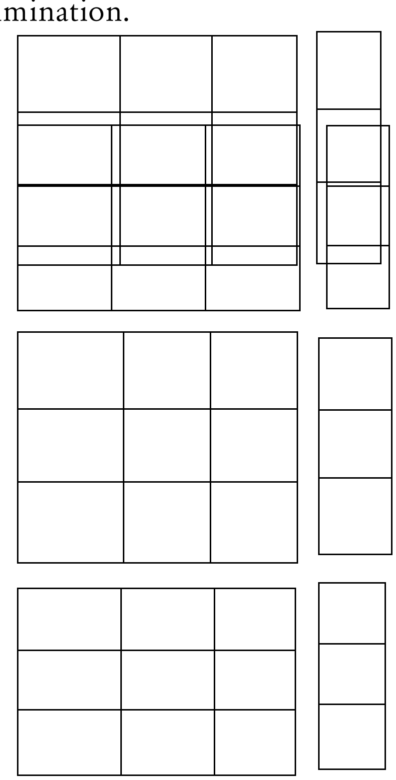
3. Show that if $Ax = b$ and $Ay = b$, then for any scalar α, $A(\alpha(y - x)) = 0$.

4. Suppose you are to solve m different linear systems of n equations in n unknowns. All of the equations have the same matrix, however, they just differ in right hand sides. Estimate how many ***additions*** are required.

5. Use the Gaussian Elimination with Partial Pivoting and Solution algorithm to solve

$$\begin{aligned} -2x_1 - 2x_2 \quad + x_3 &= 4 \\ x_1 - 4x_2 + 6x_3 &= 11 \\ 4x_1 - 8x_2 + 4x_3 &= 4 \end{aligned}$$

Show what occupies storage in the A matrix and the ip array initially and after each major step of elimination.



A

ip

6. Fill in the five blanks in the code for Gaussian Elimination with Partial Pivoting and Solution separated from elimination:

for k = 1:n
 choose ip_k such that $|A_{ip_k,k}| = \max\{|A_{i,k}| : i \geq k\}$
  if ________
   warning ('Pivot in Gaussian Elimination is zero')
  end
  swap $A_{k,k},\dots,A_{k,n}$ with $A_{ip_k,k},\dots,A_{ip_k,n}$
  for i = k+1:n
   $A_{i,k}$ = ______
   for j = ______
    $A_{i,j}$ = ___________
   end
  end
end
for k = 1:n
  swap b_k with b_{ip_k}
  for i = k+1:n
   b_i = ___________
  end
end
for i = n:-1:1
  for j = i+1:n
   $b_i = b_i - A_{i,j}x_j$
  end
  $x_i = b_i / A_{i,i}$
end