1	10	
2	10	
3	20	
4	10	
5	30	
6	20	
7	10	
Total	110	

Name ______________________________

M340L-CS Exam 1

1. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
2. Do not submit the scratch sheets. However, all of the work necessary to obtain the solution should be on these sheets.
3. Comment on all errors and omissions and enclose the **comments in boxes**

1. Either show that the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ or show that no such linear combination exists.

2. Show that if $Ax = 0$ and $Ay = b$, then for any scalar $A(y + \alpha x) = b$.

3. Find the general solutions of the systems whose **augmented** matrix is:

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$$

4. Suppose the application of the Gaussian Elimination algorithm on a 50 by 50 matrix is timed at 500 μ seconds. How much time do you estimate would be required to apply the algorithm to a 2000 by 2000 matrix? (Ignore any issues with respect to the requirement for additional memory.)

5. Use the Gaussian Elimination with Partial Pivoting and Solution algorithm to solve

$$\begin{aligned} x_1 - 6x_2 + 4x_3 &= -9 \\ 2x_1 + 6x_2 + 5x_3 &= 15 \\ 3x_1 + 6x_2 &= 9 \end{aligned}$$

Show what occupies storage in the *A* matrix and the *ip* array initially and after each major step of elimination. (Note: the 3 by 1 array below is for the ip array.)

6. Fill in the five blanks in the code for Gaussian Elimination with Partial Pivoting and Solution

for k = 1:n
    choose ip_k such that $|A_{ip_k,k}| = \max\{|A_{i,k}| : i \ge k\}$
    if $A_{ip_k,k}$ = 0
        warning ('Pivot in Gaussian Elimination is zero')
    end
    swap $A_{k,k},\ldots,A_{k,n}$ **with** $A_{ip_k,k},\ldots,A_{ip_k,n}$
    for i = k+*1*:n
        $A_{i,k}$ = ___________
        for j = ___________
            $A_{i,j}$ = ___________
        end
    end
end
for k = 1:n
    swap b_k **with** b_{ip_k}
    for i = k+1:n
        b_i = ___________
    end
end
for i = n:-1:1
    for j = i+1:n
        $b_i = b_i - A_{i,j}x_j$
    end
    x_i = ___________
end

7. Answer T(rue) or F(alse):

____a. A consistent system of linear equations could have no solution.

____b. The column space of A is the set of all solutions of $Ax = b$.

____c. If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $Ax = b$ is consistent for each b in $\mathbb{R}^m$.

____d. If A and B are $m \times n$, then AB^T is defined but $A^T B$ is not defined.

____e. If $AC = 0$, then either $A = 0$ or $C = 0$.

____f. The column space of A is the set of all vectors that can be written as Ax for some x.

____g. If the problem $Ax = b$ has a solution x, then the problem $HAx = Hb$ has the same solution x, for any matrix H (for which HA and Hb are defined.)

____h. Let matrix A have columns $\begin{bmatrix} A_{\cdot,1} & \cdots & A_{\cdot,n} \end{bmatrix}$ and the problem $Ax = b$ has a solution x, then $b = x_1 A_{\cdot,1} + \ldots + x_n A_{\cdot,n}$.

____i. If the $n \times n$ matrix A has an inverse B then $AB = I$ but BA does not necessarily equal I.

____j. _ For some $n \times n$ matrices A and B, $AB \neq BA$.