# Exam Review Problems

**1. Finding inverses:**

**a.** Determine the inverse of $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & 5 \\ 0 & -2 & 2 \end{bmatrix}$.

**b.** Determine the inverse of the elementary row operation $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix}$. (You can do all the arithmetic or you can ask yourself "How can I undo what $B$ does?.)

**c.** Determine the inverse of the rotation $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3/4} \\ 0 & \sqrt{3/4} & 1/2 \end{bmatrix}$. (You can do all the arithmetic or you can ask yourself "How can I undo what $R$ does?.)

**2. Finding a null space and a determinant of a 3 by 3 matrix:**

**a.** Show that $C = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & \mathbf{1} \\ 0 & -2 & 2 \end{bmatrix}$ is singular by finding a non-zero vector in its null space. (You might save yourself some work by noticing the slight difference between $C$ and $A$ from Problem 1a.)

**b.** Confirm that the determinant of $C$ is zero.

**3. Finding eigenvectors:**

Find the eigenvalues and eigenvectors of $D = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 7 & 3 \\ 0 & 0 & 2 \end{bmatrix}$.

**4. Vector Spaces:**

Identify which of the following satisfy the definition for vector spaces. For each case either mark either "Yes" or "No" in the columns "Closed under addition" and "Closed under scalar multiplication". For each answer of "No", give a simple example showing a failure of the property.

| | Closed under addition | Closed under scalar multiplication |
|---|---|---|
| **a.** The set of 4 by 2 matrices. | __________ | __________ |
| **b.** The set of 3 by 3 singular matrices. | __________ | __________ |
| **c.** The set of 3 by 3 nonsingular matrices. | __________ | __________ |
| **d.** The set of 3-vectors that are perpendicular to **both** [1, 2, 3] **and** [-1, 0, 1]. | __________ | __________ |
| **e.** The set of 3-vectors that are perpendicular to **either** [1, 2, 3] **or** [-1, 0, 1]. | __________ | __________ |

**5. Column Space:**

**a.** How many linearly independent vectors are in the column space of matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & 5 \\ 0 & -2 & 2 \end{bmatrix}?$$

**b.** How many linearly independent vectors are in the column space of matrix

$$C = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & \mathbf{1} \\ 0 & -2 & 2 \end{bmatrix}?$$

**c.** For any matrix $E$ and vector $b$, describe a solution of $Ex = b$ in terms of the column space of $E$.

**6. Proofs:**

**a.** Prove that if $y$ is an eigenvector of $F$ then $y$ is also an eigenvector of $6F^3 - 2F + 3I$. (Hint: If $Fy = ay$, what is $F^2$ ? What is $cF^2$ ?

**b.** Prove that if an inverse matrix $A^{-1}$ exists so that $A^{-1}A = I$ then the null space of $A$ contains only the zero vector.

**c.** Prove for any $n$ by $n$ matrices $G$ and $H$ (where $H$ is non-singular) that if $\lambda$ is an eigenvalue of $G$ with corresponding eigenvector $v$ then $\lambda$ is an eigenvalue of $HGH^{-1}$ with corresponding eigenvector $Hv$.

**7. Gram-Schmidt and Least Squares:**

**a.** Using the Gram-Schmidt Algorithm transform the matrix $M = \begin{bmatrix} 1 & 3 & 4 \\ 0 & -2 & 5 \\ 4 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$ into a product $M = QR$, where $Q^T Q = I$ and $R$ is upper triangular.

**b.** Using this, for $b = \begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \end{bmatrix}$, determine $x$ that minimizes $\|Mx - b\|$. .