Gaussian Elimination Algorithm
with Partial Pivoting and Elimination Separated from Solving

Forward Elimination Applied to Matrix

for \( k = 1:n \)  
choose \( ip_k \) such that \( |A_{ip_k,k}| = \max\{|A_{i,k}| : i \geq k \} \)  
if \( A_{ip_k,k} = 0 \)  
warning ('Pivot in Gaussian Elimination is zero')  
and maybe get out of here  
end  
swap \( A_{k,k}, \ldots, A_{k,n} \) with \( A_{ip_k,k}, \ldots, A_{ip_k,n} \)  
for \( i = k+1:n \)  
\( A_{i,k} = A_{i,k} / A_{k,k} \)  
for \( j = k+1:n \)  
\( A_{i,j} = A_{i,j} - A_{i,k} A_{k,j} \)  
end  
end

This results in the upper triangle of the eliminated system in the upper triangle of \( A \), the multipliers in the strict lower triangle of \( A \), and the swapping information in the \( ip \) array.

Solving  
notice no appearance of \( b \) until now

for \( k = 1:n \)  
swap \( b_k \) with \( b_{ip_k} \)  
for \( i = k+1:n \)  
\( b_i = b_i - A_{i,k} b_k \)  
end

for \( i = n:-1:1 \)  
for \( j = i+1:n \)  
\( b_i = b_i - A_{i,j} x_j \)  
end  
\( x_i = b_i / A_{i,i} \)  
end

and the output is the solution \( x \).