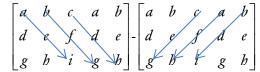
M 340L - CS Homework Set 10

1. 1. Calculate the determinants of

a.
$$\begin{bmatrix} 3 & 6 \\ -1 & 4 \end{bmatrix}$$

b.
$$\begin{bmatrix} 2 & 2 & 4 \\ -2 & 0 & 3 \\ 4 & 3 & -1 \end{bmatrix}$$

2. The expansion of a 3 x 3 determinant can be remembered by the following device. Add a copy of the first two columns to the right of the matrix, and compute the determinant adding the products along the northwest-to-southeast diagonals and subtracting the products along the northeast-to-southwest diagonals:



Use this method to compute the determinants:

a.
$$\begin{bmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{bmatrix}$$
 b.
$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

- 3. Prove that for an invertible matrix A, $det(A^{-1}) = 1/det(A)$. (Hint: Remember $AA^{-1} = I$.)
- **4.** Answer true or false to the following. If false offer a counterexample.
 - a. If the columns of A are linearly dependent, then det(A) = 0.
 - b. det(A + B) = det(A) det(B).
- c. The determinant of A is the product of the diagonal entries in A.
- d. If det(A) is zero, then two rows or two columns are the same, or a row or a column is zero.

- 5. Answer true or false to the following. If false offer a counterexample.
- a. If $Ax = \lambda x$ for some scalar λ , then x is an eigenvector of A.
- b. If v1 and v2 are linearly independent eigenvectors, then they correspond to distinct eigenvalues.
- c. The eigenvalues of a matrix are on its main diagonal.
- 6. For each of these matrices,
 - find the characteristic polynomial $p(\lambda) = \det(A \lambda I)$.
 - factor it to get the eigenvalues: $\lambda_1, \lambda_2, ..., \lambda_n$.
 - for i=1,...,n: find x^i the eigenvector corresponding λ_i . (that is, find a vector x^i in the nullspace of $A-\lambda_i I$).
 - Scale all eigenvectors so the largest component is + 1.

$$\mathbf{a.} \ \ A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}.$$

b.
$$A = \begin{bmatrix} 8 & -12 & 0 \\ -3 & 9 & -3 \\ -5 & 3 & 3 \end{bmatrix}$$
.

7. Two eigenvectors of an upper triangular matrix:

Let
$$U = \begin{bmatrix} a & b & \cdots \\ 0 & c & \cdots \\ 0 & 0 & \ddots \end{bmatrix}$$
 be *n* by *n* and upper triangular. Assume $a \neq c$.

- a. Show that the eigenvector corresponding to the eigenvalue a is e_1 (i.e the first column of the n by n identity matrix).
- **b.** Show that the eigenvector corresponding to the eigenvalue c is $\begin{vmatrix} b/(c-a) \\ 1 \\ 0 \\ \vdots \end{vmatrix}$.