1. 1. Calculate the determinants of
   a. \[
   \begin{bmatrix} 3 & 6 \\ -1 & 4 \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix} 2 & 2 & 4 \\ -2 & 0 & 3 \\ 4 & 3 & -1 \end{bmatrix}
   \]

2. The expansion of a 3 x 3 determinant can be remembered by the following device. Add a copy of the first two columns to the right of the matrix, and compute the determinant adding the products along the northwest-to-southeast diagonals and subtracting the products along the northeast-to-southwest diagonals:

   \[
   \begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & b & i & g & b \end{bmatrix}
   \]

   Use this method to compute the determinants:
   a. \[
   \begin{bmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}
   \]

3. Prove that for an invertible matrix \( A \), \( \det(A^{-1}) = 1/\det(A) \). (Hint: Remember \( AA^{-1} = I \).)

4. Answer true or false to the following. If false offer a counterexample.
   a. If the columns of \( A \) are linearly dependent, then \( \det(A) = 0 \).

   b. \( \det(A + B) = \det(A)\det(B) \).

   c. The determinant of \( A \) is the product of the diagonal entries in \( A \).

   d. If \( \det(A) \) is zero, then two rows or two columns are the same, or a row or a column is zero.
5. Answer true or false to the following. If false offer a counterexample.

a. If \( Ax = \lambda x \) for some scalar \( \lambda \), then \( x \) is an eigenvector of \( A \).

b. If \( v_1 \) and \( v_2 \) are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

c. The eigenvalues of a matrix are on its main diagonal.

6. For each of these matrices,
   - find the characteristic polynomial \( p(\lambda) = \det(A - \lambda I) \).
   - factor it to get the eigenvalues: \( \lambda_1, \lambda_2, \ldots, \lambda_n \).
   - for \( i = 1, \ldots, n \): find \( x^i \) the eigenvector corresponding \( \lambda_i \); (that is, find a vector \( x^i \) in the nullspace of \( A - \lambda_i I \)).
   - Scale all eigenvectors so the largest component is +1.

   a. \( A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \).

   b. \( A = \begin{bmatrix} 8 & -12 & 0 \\ -3 & 9 & -3 \\ -5 & 3 & 3 \end{bmatrix} \).

7. Two eigenvectors of an upper triangular matrix:

Let \( U = \begin{bmatrix} a & b & \cdots \\ 0 & c & \cdots \\ 0 & 0 & \ddots \end{bmatrix} \) be \( n \) by \( n \) and upper triangular. Assume \( a \neq c \).

   a. Show that the eigenvector corresponding to the eigenvalue \( a \) is \( e_1 \) (i.e the first column of the \( n \) by \( n \) identity matrix).

   b. Show that the eigenvector corresponding to the eigenvalue \( c \) is \( \begin{bmatrix} b/(c-a) \\ 1 \\ 0 \\ \vdots \end{bmatrix} \).