M 340L – CS
Homework Set 10

1. 1. Calculate the determinants of

a. $\begin{bmatrix} 3 & 6 \\ -1 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 2 & 4 \\ -2 & 0 & 3 \\ 4 & 3 & -1 \end{bmatrix}$

2. The expansion of a 3 x 3 determinant can be remembered by the following device. Add a copy of the first two columns to the right of the matrix, and compute the determinant adding the products along the northwest-to-southeast diagonals and subtracting the products along the northeast-to-southwest diagonals:

$$\begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix} - \begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}$$

Use this method to compute the determinants:

a. $\begin{bmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}$

3. Prove that for an invertible matrix A, $\det(A^{-1}) = 1/\det(A)$. (Hint: Remember $AA^{-1} = I$.)

4. Answer true or false to the following. If false offer a counterexample.

a. If the columns of A are linearly dependent, then $\det(A) = 0$.

b. $\det(A + B) = \det(A)\det(B)$.

c. The determinant of A is the product of the diagonal entries in A.

d. If $\det(A)$ is zero, then two rows or two columns are the same, or a row or a column is zero.

5. Answer true or false to the following. If false offer a counterexample.

a. If $Ax = \lambda x$ for some scalar λ, then x is an eigenvector of A.

b. If $v1$ and $v2$ are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

c. The eigenvalues of a matrix are on its main diagonal.

6. For each of these matrices,

- find the characteristic polynomial $p(\lambda) = \det(A - \lambda I)$.
- factor it to get the eigenvalues: $\lambda_1, \lambda_2, \ldots, \lambda_n$.
- for i= $1,\ldots, n$: find x^i the eigenvector corresponding λ_i.(that is, find a vector x^i in the nullspace of $A - \lambda_i I$).
- Scale all eigenvectors so the largest component is + 1.

a. $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$.

b. $A = \begin{bmatrix} 8 & -12 & 0 \\ -3 & 9 & -3 \\ -5 & 3 & 3 \end{bmatrix}$.

7. Two eigenvectors of an upper triangular matrix:

Let $U = \begin{bmatrix} a & b & \cdots \\ 0 & c & \cdots \\ 0 & 0 & \ddots \end{bmatrix}$ be n by n and upper triangular. Assume $a \neq c$.

a. Show that the eigenvector corresponding to the eigenvalue a is e_1 (i.e the first column of the n by n identity matrix).

b. Show that the eigenvector corresponding to the eigenvalue c is $\begin{bmatrix} b/(c-a) \\ 1 \\ 0 \\ \vdots \end{bmatrix}$.