M 340L – CS
Homework Set 1

Solve each system in Problems 1–6 by using elementary row operations on the equations or on the augmented matrix. Follow the systematic elimination procedure described in Lay, Section 1.1.

1.

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned}$$

2.

$$\begin{aligned} 3x_1 + 6x_2 &= -3 \\ 5x_1 + 7x_2 &= 10 \end{aligned}$$

3.

$$\begin{aligned} x_2 + 5x_3 &= -4 \\ x_1 + 4x_2 + 3x_3 &= -2 \\ 2x_1 + 7x_2 + x_3 &= -2 \end{aligned}$$

4.

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

5.

$$\begin{aligned} x_1 - 6x_2 &= 5 \\ x_2 - 4x_3 + x_4 &= 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 &= 3 \\ -x_2 + 5x_3 + 4x_4 &= 0 \end{aligned}$$

6.

$$\begin{aligned} -4x_4 &= -10 \\ 3x_2 + 3x_3 &= 0 \\ x_3 + 4x_4 &= -1 \\ -3x_1 + 2x_2 + 3x_3 + x_4 &= 5 \end{aligned}$$

7. Key statements from Lay, Section 1.1 are either quoted directly, restated slightly (but still true), or altered in some way that makes them false in some cases. Mark each statement True or False, and justify your answer. (If true, give the approximate location where a similar statement appears, or refer to a definition or theorem. If false, give the location of a statement that has been quoted or used incorrectly, or cite an example that shows the statement is not true in all cases.)

a. Every elementary row operation is reversible.

b. A 5×6 matrix has six rows.

c. The solution set of a linear system involving variables $x_1,\ldots,x_n$ is a list of numbers $(s_1,\ldots,s_n)$ that makes each equation in the system a true statement when the values $s_1,\ldots,s_n$ are substituted for $x_1,\ldots,x_n$, respectively.

d. Two fundamental questions about a linear system involve existence and uniqueness.

e. Two matrices are row equivalent if they have the same number of rows.

f. Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

g. Two equivalent linear systems can have different solution sets.

h. A consistent system of linear equations has one or more solutions.

(8., 9. Row reduce the matrices in Problems 8 and 9 to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.)

8.

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}.$$

9.

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}.$$