M 340L – CS
Homework Set 1

Solve each system in Problems 1-6 by using elementary row operations on the equations or on the augmented matrix. Follow the systematic elimination procedure described in Lay, Section 1.1.

1.

$$\begin{aligned} x_1+5x_2&=7\\ -2x_1-7x_2&=-5 \end{aligned}$$

$$\begin{bmatrix} 1 & 5 & 7\\ -2 & -7 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 7\\ 0 & 3 & 9 \end{bmatrix}, x_2=\frac{9}{3}=3, x_1=\frac{7-5\cdot 3}{1}=-8.$$

2.

$$\begin{aligned} 3x_1+6x_2&=-3\\ 5x_1+7x_2&=10 \end{aligned}$$

$$\begin{bmatrix} 3 & 6 & -3\\ 5 & 7 & 10 \end{bmatrix} \to \begin{bmatrix} 3 & 6 & -3\\ 0 & -3 & 15 \end{bmatrix}, x_2=\frac{15}{-3}=-5, x_1=\frac{-3-6\cdot(-5)}{3}=9.$$

3.

$$\begin{aligned} x_2+5x_3&=-4\\ x_1+4x_2+3x_3&=-2\\ 2x_1+7x_2+x_3&=-2 \end{aligned}$$

$$\begin{bmatrix} 0 & 1 & 5 & -4\\ 1 & 4 & 3 & -2\\ 2 & 7 & 1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 3 & -2\\ 0 & 1 & 5 & -4\\ 2 & 7 & 1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 3 & -2\\ 0 & 1 & 5 & -4\\ 0 & -1 & -5 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 3 & -2\\ 0 & 1 & 5 & -4\\ 0 & 0 & 0 & -2 \end{bmatrix},$$

$0x_3=-2$

No solution.

4.

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -9 & 15 & -7 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix},$$

$0x_3 = 5$

No solution.

5.

$$\begin{aligned} x_1 - 6x_2 &= 5 \\ x_2 - 4x_3 + x_4 &= 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 &= 3 \\ -x_2 + 5x_3 + 4x_4 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & -1 & 5 & 4 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & -1 & 5 & 4 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 1 & 5 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & -8 \end{bmatrix},$$

$0x_4 = -8$

No solution.

6.

$$\begin{aligned} -4x_4 &= -10 \\ 3x_2 + 3x_3 &= 0 \\ x_3 + 4x_4 &= -1 \\ -3x_1 + 2x_2 + 3x_3 + x_4 &= 5 \end{aligned}$$

$$\begin{bmatrix} 0 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{bmatrix} \to \begin{bmatrix} -3 & 2 & 3 & 1 & 5 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & -4 & -10 \end{bmatrix},$$

$$x_4 = \frac{-10}{-4} = \frac{5}{2}, x_3 = \frac{-1 - 4 \cdot \frac{5}{2}}{1} = -11, x_2 = \frac{0 - 3 \cdot (-11)}{3} = 11, x_1 = \frac{5 - 2 \cdot 11 - 3 \cdot (-11) - 1 \cdot \frac{5}{2}}{-3} = -\frac{9}{2}$$

7. Key statements from Lay, Section 1.1 are either quoted directly, restated slightly (but still true), or altered in some way that makes them false in some cases. Mark each statement True or False, and justify your answer. (If true, give the approximate location where a similar statement appears, or refer to a definition or theorem. If false, give the location of a statement that has been quoted or used incorrectly, or cite an example that shows the statement is not true in all cases.)

a. Every elementary row operation is reversible.

> True: "It is important to note that row operations are reversible."

b. A 5×6 matrix has six rows.

> False: "If m and n are positive integers, an m x n matrix is a rectangular array of numbers with m rows and n columns."

c. The solution set of a linear system involving variables $x_1, \ldots, x_n$ is a list of numbers $(s_1, \ldots, s_n)$ that makes each equation in the system a true statement when the values $s_1, \ldots, s_n$ are substituted for $x_1, \ldots, x_n$, respectively.

> False: "A solution set of the system is a set of **all** lists $(s_1, \ldots, s_n)$ of numbers that makes each equation a true statement when the values $s_1, \ldots, s_n$ are substituted for $x_1, \ldots, x_n$, respectively."

d. Two fundamental questions about a linear system involve existence and uniqueness.

> True: "TWO FUNDAMENTAL QUESTIONS ABOUT A LINEAR SYSTEM
> 1. Is the system consistent; that is, does at least one solution exist?
> 2. If a solution exists, is it the only one; that is, is the solution unique?"

e. Two matrices are row equivalent if they have the same number of rows.

> False: $x_1 = 1$ and $x_1 = 0$ have the same number of rows but are not row equivalent since no elementary row operation converts the first to the second.

f. Elementary row operations on an augmented matrix never change the solution set of the associated linear system.

> True: "Suppose a system is changed to a new one via row operations. By considering each type of row operation, you can see that any solution of the original system remains a solution of the new system.

g. Two equivalent linear systems can have different solution sets.

> False: "Suppose a system is changed to a new one via row operations. By considering each type of row operation, you can see that any solution of the original system remains a solution of the new system.

h. A consistent system of linear equations has one or more solutions.

> True: "A system of linear equations is said to be consistent if it has either one solution or
> infinitely many solutions;..."

(8., 9. Row reduce the matrices in Problems 1 and 2 to reduced row echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.)

8.
$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & -2 & -8 \\ 0 & 0 & -3 & -12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} \textcircled{1} & 2 & 0 & -8 \\ 0 & 0 & \textcircled{1} & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The pivot columns are 1 and 3.

9.
$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & -3 & -6 \\ 0 & -3 & -12 & -18 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & -3 & -12 & -18 \\ 0 & 0 & -3 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} \textcircled{1} & 0 & 0 & 1 \\ 0 & \textcircled{1} & 0 & -2 \\ 0 & 0 & \textcircled{1} & 2 \end{bmatrix}$$

The pivot columns are 1, 2, and 3.