M 340L – CS
Homework Set 3

1. a. Perform the computation of $(1.3579-(975310.23/34567.89))\times.00023456$ entirely in 3 decimal digit rounding floating point.

b. What is the result of the same computation in exact arithmetic?

c. What is the error?

d. What is the (absolute) relative error?

2. **Backward Error**: The 3 decimal digit rounding floating point result of computing $(36.258-31.9876)+537.862$ is 542 . Find operands *a, b,* and *c* close to $36.258, 31.9876, \text{and } 537.862$ (relative to the largest of them), respectively, so that the exact computation of *(a-b)*+ *c* is 542. (Notice we do not compare the computed result 542 to the exact result 542.1324... .)

3. Determine if the system has a nontrivial solution. (One row operations is sufficient to determine the answer.)

$$\begin{aligned} 5x_1-3x_2+2x_3&=0\\ -3x_1-4x_2+2x_3&=0 \end{aligned}$$

4. Write the solution set of the given homogeneous system in parametric vector form. (Follow the method of Examples 1 and 2 in Lay 1.5.)

$$\begin{aligned} x_1+2x_2-3x_3&=0\\ 2x_1+x_2-3x_3&=0\\ -x_1+x_2&=0 \end{aligned}$$

In problems 5 and 6, describe all solutions of $Ax=0$ in parametric vector form, where A is row equivalent to the given matrix.

5.
$$\begin{bmatrix} 1 & -3 & -8 & 5\\ 0 & 1 & 2 & -4 \end{bmatrix}$$

6.
$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

7. Mark each statement True or False then justify your answer.

a. A homogeneous system of equations can be inconsistent.

b. If x is a nontrivial solution of $Ax = 0$, then every entry in x is nonzero.

c. The effect of adding vector p to a vector x is to move the vector x in a direction parallel to p.

d. The equation $Ax = b$ is homogeneous if the zero vector is a solution.

8. Construct a 3×3 nonzero matrix A such that the vector $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ is a solution of $Ax = 0$.

9. Given $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \\ 12 & -8 \end{bmatrix}$, find one nontrivial solution of $Ax = 0$ by inspection.

10. a. Use the **original** version of the Gaussian Elimination Algorithm to solve $\begin{bmatrix} .001 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Do this with exact arithmetic.

b. Now employ the **same** algorithm on the same problem but simulate a three decimal digit floating point environment (thus $2 + 1000$ is computed as 1000).

c. Finally, use the **same** algorithm and do the work again in a three decimal digit floating point environment (thus $-1 - .002$ is computed as -1) but swap the two rows so you are solving $\begin{bmatrix} 1 & 2 \\ .001 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.