M 340L – CS
Homework Set 6

1. Suppose P is invertible and $A = PBP^{-1}$. Solve for B in terms of P and A.

2. Suppose $(B-C)D = 0$, where B and C are $m \times n$ matrices and D is invertible. Prove that $B = C$.

3. Suppose A and B are square matrices, B is invertible, and AB is invertible. Prove that A is invertible. [Hint: Let $C = AB$, and solve this equation for A in terms of B and C.]

4. Solve the equation $AB = BC$ for A, assuming that A, B, and C are square and B is invertible.

5. Answer true or false to the following. If false offer a simple counterexample.

    a. If u and v are linearly independent, and if w is in $Span\{u, v\}$, then u, v, w are linearly dependent.

    b. If three vectors in $\mathbb{R}^3$ lie in the same plane in $\mathbb{R}^3$, then they are linearly dependent.

    c. If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

    d. If a set in $\mathbb{R}^n$ is linearly dependent then the set contains more than n vectors.

    e. If v_1 and v_2 are in $\mathbb{R}^4$ and v_2 is not a scalar multiple of v_1, then v_1, v_2 are linearly independent.

    f. If v_1, v_2, v_3 are in $\mathbb{R}^3$ and v_3 is not a linear combination of v_1, v_2, then v_1, v_2, v_3 are linearly independent.

    g. If $\{v_1, v_2, v_3, v_4\}$ is a linearly independent set of vectors in $\mathbb{R}^4$, then $\{v_1, v_2, v_3\}$ is also linearly independent.

6. Answer true or false to the following. If false offer a simple counterexample.

a. The range of the transformation $x \mapsto Ax$ is the set of all linear combinations of the columns of A.

b. Every matrix transformation is a linear transformation. c. A linear transformation preserves the operations of vector addition and scalar multiplication. d. A linear transformation $T:\mathbb{R}^n \to \mathbb{R}^m$ always maps the origin of $\mathbb{R}^n$ to the origin of $\mathbb{R}^m$.

c. A linear transformation preserves the operations of vector addition and scalar multiplication.

d. A linear transformation $T:\mathbb{R}^n \to \mathbb{R}^m$ always maps the origin of $\mathbb{R}^n$ to the origin of $\mathbb{R}^m$.

7. Answer true or false to the following. If false offer a simple counterexample.

a. If A is a 4×3 matrix, then the transformation $x \mapsto Ax$ maps $\mathbb{R}^3$ onto $\mathbb{R}^4$.

b. Every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation.

c. The columns of the standard matrix for a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ are the images under T of the columns of the $n\times n$ identity matrix .

d. A mapping $T:\mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps onto a unique vector in $\mathbb{R}^m$ (meaning two vectors in $\mathbb{R}^n$ do not map to the same vector in $\mathbb{R}^m$).

8. Find formulas for X, Y, and Z in terms of I, A, B, and C and inverses. Assume A, B, and C have inverses. (Hint: Compute the product on the left, and set it equal to the right side. First, pretend the blocks are simply real numbers but make sure you do not ever divide – you may multiply by inverses, however. Be careful about right and left multiplication.) In all cases, assume the block matrix dimensions are such that the products are defined.

a. $\begin{bmatrix} X & 0 \\ Y & Z \end{bmatrix} \begin{bmatrix} A & 0 \\ B & C \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$

b. $\begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} X & Y & Z \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$

c. $\begin{bmatrix} I & 0 & 0 \\ A & I & 0 \\ B & C & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ X & I & 0 \\ Y & Z & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$