

M 340L – CS
Homework Set 7

1. Let $u = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, $w = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$, $x = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$. Compute:

a. $w \cdot w$

b. $x \cdot w$

c. $\frac{x \cdot w}{w \cdot w}$

d. $\frac{1}{u \cdot u} u$

e. $\|x\|$

2. Find a unit vector in the direction $\begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$.

3. Find the distance between $u = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$ and $z = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$.

4. Answer true or false to the following. If false offer a simple counterexample.

a. $u \cdot v - v \cdot u = 0$.

b. For any scalar c , $\|cv\| = c\|v\|$.

c. If $\|u\|^2 + \|v\|^2 = \|u+v\|^2$, then u and v are orthogonal.

d. For an $m \times n$ matrix A and $1 \leq i \leq m$, if x is in the null space of A then x is orthogonal to A_i , the i^{th} row of A .

5. Verify the parallelogram law for vectors u and v in $\mathbb{R}^n$: $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$.

6. Given vectors u and v in $\mathbb{R}^n$, consider vectors of the form $v + \alpha u$, for all scalars α .

a. Determine α so that $v + \alpha u$, is orthogonal to u .

b. Under what circumstance is there no such α ?