

M 340L – CS
Homework Set 9

1. [15] a. A projection matrix is a matrix P such that $P^2 = P$. Using that, prove that if a (possibly rectangular) matrix Q satisfies $Q^T Q = I$, then $Q Q^T$ is a projection matrix.

b. Using the definition of an inverse, prove for any invertible matrices A and B , that $(AB)^{-1} = B^{-1}A^{-1}$.

2. [15] Given $A = \begin{bmatrix} 2 & -1 \\ 4 & 0 \\ -4 & 4 \end{bmatrix}$, use the Gram Schmidt algorithm to express $A = QR$, where

$Q^T Q = I$ and R is upper triangular.

3. [20] Identify which of the following satisfy the definition for vector spaces. For each case either mark either “Yes” or “No” in the columns “Closed under addition” and “Closed under scalar multiplication”. For each answer of “No”, give a simple example showing a failure of the property.

	Closed under addition	Closed under scalar multiplication
a. The set of two by two matrices A such that $A_{1,1}A_{2,2} \geq 0$.	_____	_____
b. The set of functions $f : \mathbb{R} \rightarrow [0,1]$ (Note: this is not $f : [0,1] \rightarrow \mathbb{R}$)	_____	_____
c. The set of 4-vectors lying on a plane in \mathbb{R}^4 that includes the origin.	_____	_____
d. The set of ordered triples of real numbers (a,b,c) so that $a \geq 0, b \geq 0$, and $c \geq 0$.	_____	_____
e. The nullspace of a 4 by 2 matrix A .	_____	_____

4. [10] Answer true or false:

_____ a. Given nonzero vectors u and v , the matrix uv^T has a null space equal to all vectors perpendicular to v .

_____ b. If both $Ax = b$ and $A\bar{x} = b$ then $x - \bar{x}$ must be in the null space of A .

_____ c. If the columns of the square matrix A form a basis for \mathbb{R}^m then A is invertible.

_____ d. For any square matrix A , if for some nonzero vector x , $Ax = 0$ then the **rows** of A are linearly dependent.

5. [10] Prove that if $A = QR$, where $Q^T Q = I$ then $\|Ax - b\| = \|Rx - Q^T b\|$.

6. [15] Find the inverse of $\begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{bmatrix}$ or show that no inverse exists.

7. [15] Suppose that applying the Gram-Schmidt algorithm to $A = \begin{bmatrix} -2 & 4 \\ 1 & 4 \\ 0 & 2 \\ 2 & -7 \end{bmatrix}$ results in

$A = QR$, where $Q = \begin{bmatrix} -2/3 & 0 \\ 1/3 & 6/7 \\ 0 & 2/7 \\ 2/3 & -3/7 \end{bmatrix}$ and $R = \begin{bmatrix} 3 & -6 \\ 0 & 7 \end{bmatrix}$. (The matrix Q satisfies $Q^T Q = I$.)

Determine x^* that minimizes $\|Ax - b\|$ over all $x \in \mathbb{R}^2$, where $b = \begin{bmatrix} 0 \\ 21 \\ 0 \\ -42 \end{bmatrix}$.

Hint: A smart student would now test $A^T r$, where $r = Ax^* - b$.