Deletions from Current CS 313H

These are the topics in my current 313H. Prior to presenting a fresh proposal for a Theory I syllabus, I thought I should indicate how I see this changing. Notice there are no details here, there are no indications of the level of knowledge, nor are there the topics to be substituted for what is being deleted. All of that begins on page 2.

1. Propositional Logic:
1.1 Axioms
1.2 Operators
1.3 Truth tables (which define the operators)
1.4 Equivalences (meaning proof by truth table)
1.5 Propositional logic proofs (much less than current, but the laws would be given and some short very formal proofs.)

2. Predicate Logic:
2.1 Quantifiers (includes translation of English into logic and back)
2.2 Generalization and Instantiation (no time for this – to be dropped)
2.3 Predicate logic proofs (no time for this – to be dropped)

3. Other Forms of Proof:
3.1 Proof by resolution (no time for this – to be dropped)
3.2 Proofs by contradiction
3.3 Inductive proofs

4. Set Theory:
4.1 Sets & Elements
4.2 Operations on Sets
4.3 Cartesian Products

5. Relations and Functions:
5.1 Relations
5.2 Properties of Relations
5.3 Functions
5.4 Properties of Functions

Important theory topics NOT included in Theory I

These are topics that I believe should be covered someplace – there's just not time in Theory I

1. Combinatorics (to be included in the Probability and Statistics course).
2. Asymptotic Dominance (Big O, little o) (to be included in 337 or the Algorithms course).
3. Program Correctness (including invariants)) (to be included in 337).
4. Recurrence Relations (to be included in 337 or the Algorithms course).
5. Searching, Sorting, and Tree Traversal (to be included in 337, Programming, or the Algorithms course).

Proposal for a Theory I Syllabus:

Time: I have specified about 30 class hours of material (actually 29-34 hours). I have not allowed for class organization, reviews, exams, exam discussion, or instructor evaluations. Also I have tried to specify only about 80% of the course – allowing the extra 20% for instructor to delve more deeply into these topics or introduce other ones.

Typical Problems: In order to indicate the depth I have suggested problems that should be solvable by the students. Almost all have been exam problems for the honors class. I estimate that an exam question for an honors class (where there are time constraints) might be a homework problem for a non-honors class. I have **not** attempted to suggest difficult questions (although you may differ).

Theoretical vs. Practical Emphasis: I have described here only the theoretical content. That is not to suggest that there be no practical content nor real word examples. For motivational purposes those elements should be included and I leave it to an instructor to select applications that are relevant. However, applications and "problem solving capabilities" should not predominate so that understanding abstraction and being able to construct simple proofs suffers. In no way should this be a cook book class of techniques for problem solving.

Order: I have suggested an order but I could imagine a rearrangement. There is always the issue of "can you discuss anything before sets?" versus "can you discuss anything before logic?". Certainly predicates are functions but nevertheless, my order introduces predicates prior to the general definition of functions. Thus, what I suggest is no more than at least one order that I believe will work. Functions could precede graphs and trees.

Topics:

1. Propositional Logic:

1.1 Axioms (meaning Principle of the Excluded Middle, etc.)
1.2 Operators
1.3 Truth tables
1.4 Equivalences

Time and Coverage: *two hours total on all of this to get to the point where the following can be solved:*

Problem:
Use a truth table to determine for which truth values of $p, q,$ and r the assertion $(r \Rightarrow q) \wedge (q \Leftrightarrow (p \vee \sim r))$ is true.

1.5 Propositional logic proofs. Modus Ponens, Modus Tolens, DeMorgan, Double Negation, plus a few more (but not as many as the 21 rules in Causey's book)

Time and Coverage: two hours to get to the point where the following can be solved:

Problem:
Using sentential calculus (with a four column format), prove that the conclusion $E \Rightarrow (J \wedge G)$ follows from premises: $E \Rightarrow (F \wedge G), G \Rightarrow (H \vee J)$, and $\sim H$.

2. Predicate Logic:
2.1 Existential and Universal Quantifiers
2.2 "Quantifier Exchange" (i.e. handling negation with quantifiers)
2.3 Understanding PL statements in English
2.4 Writing English statements in PL.

Time and Coverage: four hours for 2.1 through 2.4 to get to the point where the following can be solved:

Problems:
1. Using the predicates defined either on the set L of locations or on the set of pairs of locations:

$S(x)$	**x is in the Sky,**
$V(x,y)$	**x can be Viewed from y**
$I(x,y)$	**x is located In y,**

and the constants Austin, BeeCave, UTFootballField, UTCampus, UTTower, and MyFavoriteCloud

is it true or false that

a. $$((((\forall x \in \{clouds\})(I(x, Austin) \Rightarrow V(UTTower, x))) \wedge I(MyFavotiteCloud, Austin))) \Rightarrow (V(MyFavoriteCloud, UTTower))$$

b. $$((((\forall x \in \{clouds\})(\sim V(UTTower, x) \Rightarrow \sim I(x, Austin))) \wedge I(MyFavotiteCloud, Austin))) \Rightarrow (V(UTTower, MyFavoriteCloud))$$

2. Using the predicates defined above, express in the syntax of Predicate Calculus:

a. ***If every place located in Austin can be seen from the UTTower then every place located on the UTCampus can be seen from the UTTower.***

b. ***Nothing in BeeCave can be seen from any place in the sky.***

c. ***Each place on the UTFootballField can be seen from some place in the sky.***

3. Other Forms of Proof (including the expectations for an English proof):

3.1 Proofs by contradiction

Time and Coverage: *After sets, functions, and relations have been introduced, non-numerical examples should be used, but initially two hours to get to the point where the following can be solved:*

Problem:
There are no positive integer solutions to the equation $x^2 - y^2 = 1$. *(The students should at least be able to reproduce the argument for this.)*

3.2 Inductive proofs (including more than just examples with numbers)

Time and Coverage: *After sets, functions, and relations have been introduced, non-numerical examples should be used, but initially two hours to get to the point where the following can be solved:*

Problem:
Consider the Fibonacci sequence: $f_0 = 1, f_1 = 1, f_k = f_{k-1} + f_{k-2}$, for $k \geq 2$. Using induction, prove that for $n \geq 0$, $\sum_{k=0}^{n} f_k^2 = f_n f_{n+1}$.

4. Set Theory:

4.1 Sets & Elements (including empty sets and powers sets)
4.2 Operations on Sets

Time and Coverage: *three hours total to get to the point where the following can be solved:*

Problem:

Prove for any sets $A, B,$ and C, that if $A \subseteq C \cup B$ then $A \sim C \subseteq B$.

4.3 Cartesian Products

Time and Coverage: *two hours total to get to the point where the following can be solved:*

Problem:

Given sets $A, B,$ and C, prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

5. Relations (and Introduction to Graphs)

5. 1 Relations from a set to a set (include composition)

5. 2 Relations on a single set

Time and Coverage: *one hour to get to the point where the following can be solved:*

Problem:
Let $A = \{1,2,3,4,5\}$, $B = \{6,7,8,9\}$, $C = \{10,11,12,13\}$, and $D = \{\alpha,\beta,\gamma,\delta\}$. Further let R be a relation between A and B defined by

$$R = \{(1,7),(4,6),(5,6),(2,8)\},$$

let S be a relation between B and C defined by

$$S = \{(6,10),(6,11),(7,10),(8,13)\},$$

and let T be a relation between C and D defined by

$$T = \{(11,\beta),(10,\beta),(13,\delta),(12,\alpha),(13,\gamma)\}.$$

a. Compute the relations R^{-1} and S^{-1}.

b. Compute the relation $S \circ R$

5.3 Representation of relations by adjacency arrays and by graphs

5.4 Properties of relations (Reflexivity, Symmetry, Antisymmetry, Transitivity)

Time and Coverage: *three hours total on 5.3 and 5.4 to get to the point where the following can be solved:*

Problem:
Display the adjacency array and the graph for a relation that is:

a. Reflexive, symmetric, not antisymmetric, and transitive.
b. Not reflexive, symmetric, not antisymmetric, and not transitive.
c. Not reflexive, not symmetric, not antisymmetric, and not transitive.

Problem:
For these problems either prove the claim or give a simple counterexample. If you present a counterexample, present the relations as specific sets of ordered pairs rather than using matrices or graphs. For assume R and S are relations on a set A and $R \subseteq S$.

a If R is transitive then S is transitive.
b If S is antisymmetric then R is antisymmetric. (Note the reversal of the order from part a.)
c If R is transitive then $R \circ R$ is transitive.
d If R is symmetric then $R \circ R$ is symmetric.

5.4 Partial Orders

Time and Coverage: *one-two hours total to get to the point where the following can be solved:*

Problem:
On the set Z^+ of positive integers, consider the division relation $D = \{(x, y) : x \text{ evenly divides } y\}$. Prove that D is a partial order on Z^+.

5.5 Equivalence relations and partitions

Time and Coverage: *one-two hours total to get to the point where the following can be solved:*

Problem:
Consider the relation R on Z, the set of integers: $R = \{(x, y) : x + y \text{ is even}\}$. Prove that R is an equivalence relation.

6. Graphs
6.1 Types of graphs (e.g. directed, undirected, bipartite, complete, multigraph)
6.2 Degrees and Connectivity

Time and Coverage: *one-two hours total to get to the point where the following can be solved:*

Problem:
Let G be a simple graph with v vertices and e edges. Let m and M be the minimum and maximum degree of the vertices, respectively. Prove that $m \le \frac{2e}{v} \le M$

6.3 Eulerian and Hamiltonian graphs

Time and Coverage: *one-two hours total to get to the point where the following can be solved:*

Problems:
1. Does the following graph (picture omitted here) have an Eulerian path? Either prove it by exhibiting or prove that no such path exists.

2. Does the following graph (picture omitted here) have a Hamiltonian path? Either prove it by exhibiting or prove that no such path exists.

7. Trees
7.1 Types of trees (e.g. rooted, *m*-ary, balanced, forest)
7.2 Properties (edge-vertex-leaf relations)

Time and Coverage: *one-two hours total to get to the point where the following can be solved:*

Problems:
How many vertices and how many leaves does a complete *m*-ary tree of height *h* have?

8. Functions:

8.1 Properties of Functions (one-to-one, onto, inverses)

Time and Coverage: *three hours to get to the point where the following can be solved:*

Problem:
Given sets $A, B,$ and C and functions $f : A \to B$, and $g : B \to C$. If f and g are one-to-one then $g \circ f$ is one-to-one.

(Possible: 8.2 Pigeonhole Principle)