

**M 340L - CS**  
**Homework Set 3 Solutions**

1. a. Perform the computation of  $(1.3579 - (975310.23 / 34567.89)) \times .00023456$  entirely in 3 decimal digit rounding floating point.

$$\begin{aligned} 1.3579 &\rightarrow 1.36 \\ 975310.23 &\rightarrow 975000 \\ 34567.89 &\rightarrow 34600 \\ .00023456 &\rightarrow .000235 \end{aligned}$$

$$\begin{aligned} 975000 / 34600 &= 28.179190751445088... \rightarrow 28.2 \\ 1.36 - 28.2 &= -26.84 \rightarrow -26.8 \\ -26.8 \times .000235 &= -0.006298 \rightarrow -.00630 \end{aligned}$$

b. What is the result of the same computation in exact arithmetic?

$$(1.3579 - (975310.23 / 34567.89)) \times .00023456 = -0.006299446759497...$$

c. What is the error?

$$-.00630 - (-0.006299446759497...) = .0000055324...$$

d. What is the (absolute) relative error?

$$\left| \frac{.0000055324...}{-0.006299446759497...} \right| = .00008782366512143027...,$$

**2. Backward Error:** The 3 decimal digit rounding floating point result of computing  $(36.258 - 31.9876) + 537.862$  is 542. Find operands  $a$ ,  $b$ , and  $c$  close to 36.258, 31.9876, and 537.862 (relative to the largest of them), respectively, so that the exact computation of  $(a-b)+c$  is 542. (Notice we do not compare the computed result 542 to the exact result 542.1324...)

Let  $a = 36.2138$ ,  $b = 32.0317$ , and  $c = 537.8179$ , then  $(a-b)+c$  is 542 and the changes to 36.258, 31.9876, and 537.862 are about .044.

3. Determine if the system has a nontrivial solution. (One row operation is sufficient to determine the answer.)

$$\begin{aligned} 5x_1 - 3x_2 + 2x_3 &= 0 \\ -3x_1 - 4x_2 + 2x_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 5 & -3 & 2 & 0 \\ -3 & -4 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -3 & 2 & 0 \\ 0 & -29/5 & 16/5 & 0 \end{bmatrix}. \quad x_3 \text{ is free so there is a nontrivial solution.}$$

4. Write the solution set of the given homogeneous system in parametric vector form. (Follow the method of Examples 1 and 2 in Lay 1.5)

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 0 \\ 2x_1 + x_2 - 3x_3 &= 0 \\ -x_1 + x_2 &= 0 \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 1 & -3 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} &\rightarrow \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ &\rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$x_3$  is free,  $x_2 = x_3, x_1 = x_3$ . In parametric vector form this is  $x = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .

In problems 5 and 6, describe all solutions of  $Ax=0$  in parametric vector form, where  $A$  is row equivalent to the given matrix.

5.

$$\begin{bmatrix} 1 & -3 & -8 & 5 \\ 0 & 1 & 2 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -8 & 5 \\ 0 & 1 & 2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & -7 \\ 0 & 1 & 2 & -4 \end{bmatrix}. \quad x_4 \text{ is free and } x_3 \text{ is free, } x_2 = -2x_3 + 4x_4,$$

$$x_1 = 2x_3 + 7x_4. \text{ In parametric vector form this is } x = x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 4 \\ 0 \\ 1 \end{bmatrix}.$$

6.

$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 3 & 0 & 29 & -36 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 3 & 0 & 29 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$x_6 = 0, x_5$  is free,  $x_4 = -4x_5, x_3$  is free,  $x_2$  is free, and  $x_1 = 2x_2 - 3x_3 - 29x_5$ . In parametric

$$\text{vector form this is } x = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -29 \\ 0 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix}.$$

7. Mark each statement True or False then justify your answer.

a. A homogeneous system of equations can be inconsistent.

**False.** Since the zero vector is always a solution, a homogeneous system of equations can never be inconsistent.

b. If  $x$  is a nontrivial solution of  $Ax=0$ , then every entry in  $x$  is nonzero.

**False.** A nontrivial solution need only have at least one non-zero component.

c. The effect of adding vector  $p$  to a vector  $x$  is to move the vector  $x$  in a direction parallel to  $p$ .

**True.** The vertices  $0, p, x+p, x$  form a parallelogram so the effect of adding  $p$  to  $x$  to get  $x+p$  is a line parallel to the line from zero to  $p$ .

d. The equation  $Ax=b$  is homogeneous if the zero vector is a solution.

**True.** Since  $A0=b=0$ , the equation  $Ax=b$  is also  $Ax=0$  and thus is homogeneous.

8. If  $b \neq 0$ , can the solution set of  $Ax=b$  be a plane through the origin? Explain.

No, if  $b \neq 0$ , the solution set of  $Ax=b$  cannot be a plane through the origin. If it were then  $A0=0=b \neq 0$ , which is a contradiction.

9. Construct a  $3 \times 3$  nonzero matrix  $A$  such that the vector  $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$  is a solution of  $Ax=0$ .

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

10. Given  $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \\ 12 & -8 \end{bmatrix}$ , find one nontrivial solution of  $Ax=0$  by inspection.

Since twice the first column plus three times the third column is zero,  $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

11. a. Use the **original** version of the Gaussian Elimination Algorithm to solve

$$\begin{bmatrix} .001 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \text{ Do this with exact arithmetic.}$$

To solve  $\begin{bmatrix} .001 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  exactly we have for matrix A in storage

$$\left[ \begin{array}{cc|c} .001 & -1 & 1 \\ 1 & 2 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} .001 & -1 & 1 \\ 1 & 2 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} .001 & -1 & 1 \\ 1 & 1002 & -1000 \end{array} \right]$$

The solution is  $x_2 = \frac{-1000}{1002} = -.99803992\dots$ ,

$$x_1 = \frac{1 - (-1)\left(\frac{-1000}{1002}\right)}{.001} = \frac{2000}{1002} = 1.996007984\dots$$

(since  $1000 + 2$  is computed as 1000).

b. Now employ the **same** algorithm on the same problem but simulate a three decimal digit floating point environment (thus  $2 + 1000$  is computed as 1000).

To solve  $\begin{bmatrix} .001 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  we would have for matrix A in storage

$$\left[ \begin{array}{cc|c} .001 & -1 & 1 \\ 1 & 2 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -1 & 1 \\ 1 & 1000 & -1000 \end{array} \right] \text{ (since } 2 + 1000 \text{ is computed as 1000). The computed}$$

solution is  $x_2 = \frac{-1000}{1000} = -1$ , and  $x_1 = \frac{1 - (-1)(-1)}{1} = 0$ .

c. Finally, use the **same** algorithm and do the work again in a three decimal digit floating point environment (thus  $-1 - .002$  is computed as -1) but swap the two rows so you are

solving  $\begin{bmatrix} 1 & 2 \\ .001 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .

To solve  $\begin{bmatrix} 1 & 2 \\ .001 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$  with the original algorithm we would have for matrix

A in storage  $\left[ \begin{array}{cc|c} 1 & 2 & 0 \\ .001 & -1 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 2 & 0 \\ .001 & -1 & 1 \end{array} \right]$  (since  $-1 - .002$  is computed as -1). The

computed solution is  $x_2 = \frac{1}{-1} = -1$ , and  $x_1 = \frac{0 - 2(-1)}{1} = 2$ .