Workshop I - Max-k-CSP, Label Cover, Unique Label Cover

The workshop will be held on Tuesday, August 30th. There will be 5 presenters in the order given here, and each presentation should be 5 minutes. Register to lecture on August 25-26 upon announcement from the TA.

1 Max-k-CSP

An instance of Max-k-CSP(Σ) consists of a set of n variables taking values from a finite alphabet Σ along with m constraints, each of which is a relation on the assignments of k of the variables. The objective is to assign values to the variables so as to maximize the fraction of constraints satisfied.

In your presentation you should define the Max-k-CSP(Σ) problem and give interesting examples of this problem. Last, present an efficient algorithm which satisfies $\frac{1}{|\Sigma|^k}$ fraction of the optimal number of constraints.

2 Label Cover

An instance of Label Cover consists of a bipartite graph (L, R, E), a finite set of labels Σ_L for the left vertices and a finite set of labels Σ_R for the right vertices, as well as a family of functions $\{f_e : \Sigma_L \to \Sigma_R \cup \{\perp\} | \; e \in E\}$. A labelling is a pair of functions $l_L : L \to \Sigma_L$ and $l_R : R \to \Sigma_R$. An edge $e = (u, v)$ is satisfied if $f_e(l_L(u)) = l_R(v)$. Our objective is to find l_L, l_R so as to maximize the fraction of edges satisfied.

2.1 $\frac{1}{|\Sigma_R|}$-Approximation

In your presentation you should define the Label Cover problem and discuss the connection to Max-k-CSP. Last, present an efficient algorithm which satisfies $\frac{1}{|\Sigma_R|}$ fraction of the optimal number of constraints.

2.2 $\frac{d}{n}$-Approximation

Assume we are given an instance of label cover where there are n right vertices, and each of the left vertices has degree d. Present an efficient algorithm which satisfies $\frac{d}{n}$ fraction of the optimal number of constraints. Discuss the special case where the graph is a complete bipartite graph (i.e., $d = n$).

2.3 $\frac{1}{d}$-Approximation $\to (|\Sigma_R|n)^{-1/3}$-Approximation

Assume our instance is as in part 2. Present an algorithm which satisfies $\frac{1}{d}$ fraction of the optimal number of constraints. Combine the three previous algorithms, and present an algorithm which satisfies $(|\Sigma_R|n)^{-1/3}$ fraction of the optimal number of constraints. Hint: Geometric Mean

3 Unique Label Cover

An instance of Unique Label Cover consists of a bipartite graph (L, R, E), a finite set of labels Σ, and a set of 1-1 functions $\{f_e : \Sigma \to \Sigma \mid e \in E\}$. A labelling is a function $l : L \cup R \to \Sigma$. An edge $e = (u, v)$ is satisfied if $f_e(l(u)) = l(v)$. Our objective is to find a labelling so as to maximize the number of edges satisfied.

In your presentation present Unique Label Cover and show how to efficiently decide if a given instance of Unique Label Cover has a labelling which satisfies all the constraints.