CS395T: Randomized Algorithms | Problem Set No. 8
David Zuckerman | due: December 7, 2006

For this assignment, no late homeworks will be accepted.

1. Problem 6.19.

2. Problem 11.2.

3. Problem 11.7.

4. This problem shows that amplification by random walks on expanders is tight up to constant factors. Suppose there were a way to amplify the success probability of any RP algorithm as follows. If the RP algorithm uses r random bits to achieve error at most $1/2$, then there is a new RP algorithm for the same problem which uses $r + \lceil k/2 \rceil$ random bits to achieve error at most 2^{-k}, for any $k \leq 10r$. Conclude that RP = P.

5. Let G be a d-regular graph on n vertices with adjacency matrix A. Let the smallest eigenvalue of A be λ_n (so $\lambda_n < 0$).

   a) Show that for any set S of size s, the number of edges with both endpoints in S is at least
   $$\frac{1}{2}\left(\frac{ds^2}{n} + \lambda_n s\left(1 - \frac{s}{n}\right)\right).$$

   b) Conclude that the size of the largest independent set is at most $-n\lambda_n/(d - \lambda_n)$.