CS388R: Randomized Algorithms October 28, 2008
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Midterm

Instructions. There are five problems, each worth 10 points. You may assume any theorems stated in class or in the book, unless the question is to prove such a theorem. Explain all answers. You are allowed a single 8.5x11 inch, handwritten sheet of paper, with writing on both sides. You have 75 minutes.

Name: ______________________________

1. Recall that ZPP is the class of languages solvable by a randomized algorithm which runs in expected polynomial time and never errs. Show that ZPP $=$ RP $\cap$ coRP.

2. Consider a world where there are n independent securities. A dollar invested in any security has expected return μ and standard deviation σ over one time period. Since the returns are the same, investors seek to minimize risk (standard deviation). Investors A and B have the same amount of money to invest. Investor A can invest in k of these securities, whereas Investor B can invest in all n. What is the ratio of the minimum standard deviation achievable by Investor A to that of Investor B in one time period? (Hint: you may assume that the minimum occurs when the investor invests equally in all securities.)

3. Suppose you are given a randomized algorithm which, on input an undirected graph with positive integral weights on its vertices, runs in time polynomial in the sum of the weights of the vertices and outputs a set of vertices S. If the maximum weight clique is unique, then with probability at least $1/2$, S equals this maximum weight clique. Give a polynomial-time, randomized algorithm which, on input an undirected graph, outputs a maximum clique with probability at least $1/2$.

4. Give a randomized polynomial-time algorithm which takes as input n-bit integers a_i, b_i, c_i, and d_i, $i = 1, 2, \ldots, n$, and outputs whether

$$a_1^{b_1} + a_2^{b_2} + \ldots + a_n^{b_n} = c_1^{d_1} + c_2^{d_2} + \ldots + c_n^{d_n}.$$

(First observe why you can't just multiply it out.)

5. Show that with probability at least $1-2^{-\Omega(n)}$, a random multigraph on n vertices with $10n$ edges has at least $\Omega(n)$ isolated vertices. (A random multigraph is obtained by choosing the edges randomly, with replacement, so there may be multiple edges. An isolated vertex is a vertex of degree 0. You may use the inequality $(1-1/k)^k \geq 1/3$ for $k \geq 6$.)