CS388R: Randomized Algorithms
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Problem Set No. 3
due: Oct. 2, 2008

1. Problem 4.11. Randomly rounding a real number r to an integer means rounding to $\lceil r \rceil$ with probability $r - \lfloor r \rfloor$, and to $\lfloor r \rfloor$ otherwise. Randomly rounding a vector means randomly rounding its coordinates independently.

2. Problem 4.15.

3. Problem 4.16.

4. Let $\mathcal{D}_i$, $i = 1, 2, \ldots, n$, be finite sets. Suppose f satisfies the Lipschitz condition (Definition 4.12). Prove that if all X_i are chosen uniformly and independently at random from $\mathcal{D}_i$, then

$$\Pr[f(X_1, X_2, \ldots, X_n) - \mathrm{E}[f(X_1, X_2, \ldots, X_n) \geq \lambda\sqrt{n}] \leq e^{-\lambda^2/2}.$$

5. Consider using randomized routing in an N-node hypercube to route at most ℓN packets, where each node is the source of at most ℓ packets and the destination of at most ℓ packets. Give a high probability bound on the time for all packets to reach their destination. (This should be better than ℓ times the time for N packets; ℓ is not necessarily a constant.)