CS388R: Randomized Algorithms — Problem Set No. 4

David Zuckerman — due: Oct. 14, 2008

1. Problem 5.10

2. Give a deterministic algorithm which, on input m clauses, each the OR of k distinct literals, outputs an assignment satisfying at least $m(1 - 2^{-k})$ clauses.

3. Show that BPP has polynomial-sized circuits.

4. Give a randomized algorithm which, on input a directed graph $G = (V, E)$ with at least one simple k-path, outputs such a path. (A simple path has no repeat vertices, and a k-path is a path of length k, i.e., it has k edges.) Your algorithm should run in expected time $2^{O(k)}|E|$. Hint: you may use the fact that such an algorithm exists (even a deterministic one) when, in addition, the vertices are colored with $k + 1$ colors, and at least one k-path has its $k + 1$ vertices assigned all different colors.

5. The sequence $A = (a_1, a_2, \ldots, a_\ell)$ of elements in $[n]$ is given in order one element at a time. Your task is to approximate $z = \sum_{i=1}^{n} m_i^2$, where $m_i = |\{j | a_j = i\}|$; the difficulty is that you are allowed space only $O(\log^2(\ell n))$. Give a randomized algorithm which, with probability at least .9, has relative error at most .1. You may assume that for any $r \geq s$, there is a pseudorandom generator G mapping an $O(\log^2 r)$-bit seed to $r \times s$ matrices with ± 1 entries such that random projections with such pseudorandom matrices obey the Johnson-Lindenstrauss Theorem. You may further assume that for any (i, j) and seed x, G requires space $O(\log r)$ to compute the (i, j)th entry of the matrix $G(x)$.