CS395T: Pseudorandomness and Combinatorial Constructions — Problem Set No. 3
David Zuckerman — due: March 8

1. (10 points) Let G be a d-regular graph with adjacency matrix A.

   a) Prove that A has no eigenvalue larger than d in absolute value.

   b) Prove that if G is not connected, then the multiplicity of the d eigenvalue is at least 2.

2. (10 points) Let G be a d-regular graph on n vertices with adjacency matrix A. Let the smallest eigenvalue of A be λ_n (so $\lambda_n < 0$). Let $E(S,U)$ denote the number of edges between sets S and U.

   a) Show that for any set S of size s,

$$E(S,S) \geq \frac{ds^2}{n} + \lambda_n s \left(1 - \frac{s}{n}\right).$$

   b) Show that the size of the largest independent set is at most $-n\lambda_n/(d-\lambda_n)$.

3. (15 points) Generalize the random walk-based amplification of RP to BPP. In particular, show that if there is a BPP algorithm using r random bits to achieve a sufficiently small constant error, then the amplification based on a random walk on an expander can achieve error 2^{-s} using $r + O(s)$ random bits.