CS 311H: Discrete Math (Fall 2026)

Problem Set #1

Due: September 4, 2026

Problem 1: Equivalence using Truth Tables

Use truth tables to show that the following statements are logically equivalent.

(a) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

Solution.

p	q	r	$(p \to q)$	$(p \to r)$	$(p \to q) \vee (p \to r)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$(q \vee r)$	$(p \to (q \vee r))$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

The truth values in the columns $(p \to q) \vee (p \to r)$ and $(p \to (q \vee r))$ are the same. Therefore, the two compound propositions are equivalent.

(b) $\neg((p \vee q) \to \neg q)$ and q

Solution.

p	q	$\neg q$	$(p \vee q)$	$(p \vee q) \rightarrow \neg q)$	$\neg((p \vee q) \rightarrow \neg q)$
T	T	F	T	F	T
T	F	T	T	T	F
F	T	F	T	F	T
F	F	T	F	T	F

The truth values in the columns q and $\neg((p \vee q) \rightarrow \neg q)$ are the same. Therefore the two compound propositions are equivalent. Therefore, $\neg((p \vee q) \rightarrow \neg q) \equiv q$.

Problem 2: Equivalence-style Proof

Using the basic equivalances, prove that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology.

Note: Please show all steps (including associativity and commutativity). Please use parenthesis, do not rely on order of operations.

Solution.

$$
\begin{aligned}
& (p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r) \\
\equiv\ & < \text{Implication Simplification} > \\
& \neg\left((p \vee q) \wedge (\neg p \vee r)\right) \vee (q \vee r) \\
\equiv\ & < \text{DeMorgan's} > \\
& (\neg (p \vee q) \vee \neg (\neg p \vee r)) \vee (q \vee r) \\
\equiv\ & < \text{DeMorgan's} \times 2 > \\
& ((\neg p \wedge \neg q) \vee (\neg(\neg p) \wedge \neg r)) \vee (q \vee r) \\
\equiv\ & < \text{double negation} > \\
& ((\neg p \wedge \neg q) \vee (p \wedge \neg r)) \vee (q \vee r) \\
\equiv\ & < \text{associativity} > \\
& (((\neg p \wedge \neg q) \vee (p \wedge \neg r)) \vee q) \vee r \\
\equiv\ & < \text{associativity} > \\
& ((\neg p \wedge \neg q) \vee ((p \wedge \neg r) \vee q)) \vee r \\
\equiv\ & < \text{commutativity} > \\
& ((\neg p \wedge \neg q) \vee (q \vee (p \wedge \neg r))) \vee r \\
\equiv\ & < \text{associativity} \times 2 > \\
& ((\neg p \wedge \neg q) \vee q) \vee ((p \wedge \neg r) \vee r) \\
\equiv\ & < \text{distributivity} \times 2 > \\
& ((\neg p \vee q) \wedge (\neg q \vee q)) \vee ((p \vee r) \wedge (\neg r \vee r)) \\
\equiv\ & < \text{Excluded Middle} \times 2 > \\
& ((\neg p \vee q) \wedge T) \vee ((p \vee r) \wedge T) \\
\equiv\ & < \text{Identity laws} > \\
& (\neg p \vee q) \vee (p \vee r) \\
\equiv\ & < \text{Associativity} > \\
& ((\neg p \vee q) \vee p) \vee r \\
\equiv\ & < \text{Associativity} > \\
& (\neg p \vee (q \vee p)) \vee r \\
\equiv\ & < \text{Commutativity} > \\
& (\neg p \vee (p \vee q)) \vee r \\
\equiv\ & < \text{Associativity} > \\
& ((\neg p \vee p) \vee q) \vee r \\
\equiv\ & < \text{Associativity} > \\
& (\neg p \vee p) \vee (q \vee r) \\
\equiv\ & < \text{Excluded Middle} > \\
& T \vee (q \vee r) \\
\equiv\ & < \text{Domination laws} > \\
& T
\end{aligned}
$$

$\therefore ((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology.

Problem 3: System specification consistent or not?

Given a system, its specifications must be **consistent**, that is they should not contain conflicting requirements that could be used to derive a contradiction. When a system specification is inconsistent, there would be no way to develop a system that satisfies all the specifications.

Another way to think about system specification consistency, the conjunction of all the specifications must be satisfiable.

Are these system specification consistent? Justify your answer.

1. The system is in multiuser state if and only if it is operating normally.
2. If the system is operating normally, the kernel is functioning.
3. The kernel is not functioning or the system is in interrupt mode.
4. If the system is not in multiuser state, then it is in interrupt mode.
5. The system is not in interrupt mode.

Solution. Let

p **:** "The system is in multiuser state"

q **:** "The system is operating normally"

r **:** "The kernel is functioning"

s **:** "The system is in interrupt mode"

The system specifications states that

1. $p \leftrightarrow q$
2. $q \rightarrow r$
3. $\neg r \vee s$
4. $\neg p \rightarrow s$
5. $\neg s$

For the system to be consistent, all the specifications should not contain conflicting requirements. Starting with 5, for this proposition to be true, $\neg s \equiv T$ or in other words, $s \equiv F$.

For 4 to be true, from the above we have $s \equiv F$, therefore $\neg p$ must be false. If $\neg p$ were not false, then the implication would not hold.

Given $p \equiv T$, for 1 to be true, q must also be true. Now we have $q \equiv T$.

Given $q \equiv T$,for 2 to be true, r must also be true. Now we have $r \equiv T$.

Given $r \equiv T$, for 3 to be true, s must also be true. This contradicts our original truth value assignment of s.

Therefore, the system is inconsistent.

Problem 4: Prove using the Rules of Inference

$$\begin{array}{rl} & R \to W \\ & W \to S \\ & S \to C \\ & R \\ & C \to I \\ \hline \therefore & I \end{array}$$

Solution.

1.	$R \to W$	premise
2.	$W \to S$	premise
3.	$S \to C$	premise
4.	R	premise
5.	$C \to I$	premise
6.	$R \to S$	Hypothetical syllogism [1],[2]
7.	$R \to C$	Hypothetical syllogism [6],[3]
8.	$R \to I$	Hypothetical syllogism [7],[5]
9.	I	Modus Ponens [8], [4]

Problem 5: Prove using the Rules of Inference

$$\begin{array}{rl} & C \vee P \\ & C \to W \\ & W \to X \\ & \neg X \\ \hline \therefore & P \end{array}$$

Solution.

1.	$C \vee P$	premise
2.	$C \to W$	premise
3.	$W \to X$	premise
4.	$\neg X$	premise
5.	$\neg W$	Modus Tollens [4],[3]
6.	$\neg C$	Modus Tollens [5],[2]
7.	P	Disjunctive Syllogism [6],[1]

Problem 6: Prove using the Rules of Inference

$$
\begin{array}{rl}
 & (a \wedge q) \rightarrow m \\
 & f \rightarrow q \\
 & \neg p \rightarrow a \\
\hline
\therefore & \neg m \rightarrow (\neg f \vee p)
\end{array}
$$

Solution.

1.	$(a \wedge q) \rightarrow m$	premise
2.	$f \rightarrow q$	premise
3.	$\neg p \rightarrow a$	premise
4.	$\neg(a \wedge q) \vee m$	Implication Simplification [1]
5.	$(\neg a \vee \neg q) \vee m$	DeMorgan's [4]
6.	$\neg a \vee (\neg q \vee m)$	Associativity [5]
7.	$\neg a \vee (m \vee \neg q)$	Commutativity [6]
8.	$(\neg a \vee m) \vee \neg q$	Associativity [7]
9.	$\neg f \vee q$	Implication Simplification [2]
10.	$(\neg a \vee m) \vee \neg f$	Resolution [9],[8]
11.	$\neg a \vee (m \vee \neg f)$	Associativity [10]
12.	$p \vee a$	Implication Simplification; Double Negation [3]
13.	$(m \vee \neg f) \vee p$	Resolution [12][11]
14.	$m \vee (\neg f \vee p)$	Associativity [13]
15.	$\neg m \rightarrow (\neg f \vee p)$	Implication Simplification [14]