CS 311H: Discrete Math (Fall 2026)

Problem Set #2

Due: Sep 11, 2026

Problem 1: Translating English to Nested Quantifiers

Let $L(x, y)$ be the predicate "x loves y. where the domain for both x and y consists of all people in the world D_w. Use quantifiers to express each of these statements.

(a) Everybody loves Jerry.

Solution. $(\forall x | x \in D_w : L(x, \text{Jerry}))$

(b) Everybody loves somebody.

Solution. $\big(\forall x | x \in D_w : (\exists y | y \in D_w : L(x, y))\big)$

(c) There is somebody whom everybody loves.

Solution. $\big(\exists y | y \in D_w : (\forall x | x \in D_w : L(x, y))\big)$

(d) Nobody loves everybody.

Solution. $\neg \big(\exists x | x \in D_w : (\forall y | y \in D_w : L(x, y))\big)$ or $\big(\forall x | x \in D_w : (\exists y | y \in D_w : \neg L(x, y))\big)$

Note: The way to think about this problem is to think of what "Nobody" means, or rather how to express this sentence using a negation. Another way to think of this sentence is "It is not the case that somebody loves everybody" or "There does not exist anyone that loves everybody". This helps us write out the quantifier.

(e) There is somebody whom Lydia does not love.

Solution. $(\exists y | y \in D_w : \neg L(\text{Lydia}, y))$

(f) There is somebody whom no one loves.

Solution. $\left(\exists y | y \in D_w : \left(\forall x | x \in D_w : \neg L(x, y)\right)\right)$

Note: Another way to express this sentence is to say "There is somebody such that everyone does not love this person."

(g) There is exactly one person whom everybody loves.

Solution. $\left(\exists y | y \in D_w : \left(\forall x | x \in D_w : L(x, y)\right) \wedge \left(\forall z | z \in D_w : \left(\forall w | w \in D_w : L(w, z)\right) \rightarrow z = y\right)\right)$

Note: This is a tricky one. Let's break it down. "There is exactly one person whom everybody loves." To express this sentence in quantifiers, we first have to establish that there first exists someone, let's call this person y. We must then establish that everyone loves this person y. In addition to this quantifer (that is why the conjunction), we have to establish that this person y is unique (that is there is just one such person). To express that that this person y is unique, we can say that for every person (for all z) if everyone (for all w) loves z then this z must be the same person as y.

(h) There is exactly two people that Lynn loves.

Solution.
$\left(\exists x | x \in D_w : \left(\exists y | y \in D_w : (x \neq y) \wedge L(\text{Lynn}, x) \wedge L(\text{Lynn}, y) \wedge \left(\forall z | z \in D_w : L(\text{Lynn}, z) \rightarrow (z = x \vee z = y)\right)\right)\right)$

Note: Yet another tricky one. Let's break it down. "There is exactly two people that Lynn loves." To express this sentence in quantifiers, we first have to establish that two people (say x and y) that Lynn loves, and that x and y is not the same person. In addition (that's why the conjunction), we have to establish that everyone (for all z) if Lynn loves this z, then z must either be x or y.

(i) Everyone loves himself or herself.

Solution. $\left(\forall x | x \in D_w : L(x, x)\right)$

(j) There is someone who loves no one besides himself or herself.

Solution. $\left(\exists x | x \in D_w : \left(\forall y | y \in D_w : L(x, y) \leftrightarrow x = y\right)\right)$ or
$\left(\exists x | x \in D_w : L(x, x) \wedge \left(\forall y | y \in D_w : \left(L(x, y) \rightarrow x = y\right)\right)\right)$.
Note: Why are these two equivalent?

$$\Big(\exists x | x \in D_w : \big(\forall y | y \in D_w : L(x, y) \leftrightarrow x = y\big)\Big)$$

$\equiv <$ Bicondtional $>$

$$\Bigg(\exists x | x \in D_w : \Bigg(\forall y | y \in D_w : \big(L(x, y) \rightarrow x = y\big) \wedge \big(x = y \rightarrow L(x, y)\big)\Bigg)\Bigg)$$

$\equiv <$ Since $x = y >$

$$\Bigg(\exists x | x \in D_w : \Bigg(\forall y | y \in D_w : \big(L(x, y) \rightarrow x = y\big) \wedge \big(L(x, x)\big)\Bigg)\Bigg)$$

$\equiv <$ Manipulating the quantifiers, since $L(x, x)$ does not depend on $y >$

$$\Big(\exists x | x \in D_w : L(x, x) \wedge \big(\forall y | y \in D_w : L(x, y) \rightarrow x = y\big)\Big)$$

Problem 2: Not Mother of Self

Let x and y be in the domain of all living things L. Let $M(x, y)$ be the predicate x is the mother of y. Prove the following:

$$\begin{array}{rl} & \Big(\forall x | x \in L : \big(\forall y | y \in L : M(x,y) \rightarrow \neg M(y,x)\big)\Big) \\ \hline \therefore & \big(\forall x | x \in L : \neg M(x,x)\big) \end{array}$$

Solution.

1.	$\Big(\forall x \mid x \in L : \big(\forall y \mid y \in L : M(x,y) \rightarrow \neg M(y,x)\big)\Big)$	premise
2.	$\big(\forall y \mid y \in L : M(c,y) \rightarrow \neg M(y,c)\big)$	Universal Instantiation [1]
3.	$M(c,c) \rightarrow \neg M(c,c)$	Universal Instantiation[1] [2]
4.	$\neg M(c,c) \vee \neg M(c,c)$	Implication simplification [3]
5.	$\neg M(c,c)$	Idempotent laws [4]
6.	$\big(\forall x \mid x \in L : \neg M(x,x)\big)$	Universal generalization [5]

Problem 3: Prove using Rules of Inference

$$
\begin{array}{rl}
& \big(\exists y | y \in D : P(y)\big) \rightarrow \big(\forall x | x \in D : Q(x)\big) \\
& P(a) \\
& R(c) \rightarrow \neg \big(P(a) \wedge Q(b)\big) \\
\hline
\therefore & \neg \big(\forall z | z \in D : R(z)\big)
\end{array}
$$

Solution.

1.	$\big(\exists y \mid y \in D : P(y)\big) \rightarrow \big(\forall x \mid x \in D : Q(x)\big)$	premise
2.	$P(a)$	premise
3.	$R(c) \rightarrow \neg \big(P(a) \wedge Q(b)\big)$	premise
4.	$\big(\exists y \mid y \in D : P(y)\big)$	Existential generalization [2]
5.	$\big(\forall x \mid x \in D : Q(x)\big)$	Modus ponens [1][4]
6.	$Q(b)$	Universal Initialization [5]
7.	$P(a) \wedge Q(b)$	Conjunction [2],[6]
8.	$\neg R(c)$	Modus tollens p[7],[3]
9.	$\big(\exists z \mid z \in D : \neg R(z)\big)$	Existential generalization [8]
10.	$\neg \big(\forall z \mid z \in D : R(z)\big)$	Negating quantifiers [9]

Problem 4: Prove using the Rules of Inferences

$$\begin{array}{rl} & \Big(\forall y \mid y \in D : \big((\exists x \mid x \in D : F(x,b)) \rightarrow R(b,y)\big)\Big) \\ \hline \therefore & (\forall x \mid x \in D : F(a,x)) \rightarrow \Big(\forall x \mid x \in D : (\exists y \mid y \in D : R(y,x))\Big) \end{array}$$

Solution.

1.	$\Big(\forall y \mid y \in D : \big((\exists x \mid x \in D : F(x,b)) \rightarrow R(b,y)\big)\Big)$	Premise
2.	$(\forall x \mid x \in D : F(a,x))$	Conditional Premise
3.	$F(a,b)$	Universal Instantiation [2]
4.	$(\exists x \mid x \in D : F(x,b))$	Existential Generalization [3]
5.	$(\exists x \mid x \in D : F(x,b)) \rightarrow R(b,c)$	Universal Instantiation [1]
6.	$R(b,c)$	Modus Ponens [4][5]
7.	$(\exists y \mid y \in D : R(y,c))$	Existential Generalization [6]
8.	$\Big(\forall x \mid x \in D : (\exists y \mid y \in D : R(y,x))\Big)$	Universal Generalization [7]
9.	$(\forall x \mid x \in D : F(a,x)) \rightarrow \Big(\forall x \mid x \in D : (\exists y \mid y \in D : R(y,x))\Big)$	Conditional Discharge[8][2]