

CS 311H: Discrete Math (Fall 2026)

Problem Set #3

Due: Sep 18, 2026

Problem 1: Prove

Prove that if m and n are integers, and mn is even, then m is even or n is even. Clearly state the proof method you choose to use.

Problem 2: Prove

Prove that for an integer n if n^2 is odd, then n is odd.

Problem 3: Prove

For any integer x , if x is divisible by 3, then x^2 is divisible by 3.

Problem 4: Prove by contradiction

Prove that $\sqrt{3}$ is irrational.

Hint: You may use the following result without proving it in your proof “For any integer x , if 3 is a factor of x^2 , then 3 is a factor of x ”.

Problem 5: [x points] **Proof by Cases**

Prove that for any odd integer n , there exists an integer m such that n^2 can be written as $8m + 1$.

Hint: You may use the result that “for any integer x , there exists an integer t , such that $x = 4t, 4t + 1, 4t + 2$, or $4t + 3$ ” without proving it.

Additional Hint: If an integer x , can be written as $x = 4t, 4t + 1, 4t + 2$, or $4t + 3$ for some integer t , which of these four forms will be odd? (You do not need to prove this before using it in your proof.) You may want to use these as your cases.

Problem 6:

Suppose a, b are integers. Prove that $(a - 3)b^2$ is even if and only if a is odd or b is even.